

Mathematical modeling and analysis of corruption dynamics

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Abstract

We propose a mathematical model for corruption by considering awareness created by anti-corruption and counseling in jail. The model is proved to be both epidemiologically and mathematically well posed. We showed that all solutions of the model are positive and bounded with initial conditions in a certain meaningful set. The existence of unique corruption free and endemic equilibrium points are investigated and the basic reproduction number is computed. Then, we study the local asymptotic stability of these equilibrium points. The analysis shows that the system has a locally asymptotically stable corruption-free equilibrium point when the reproduction number is less than one and locally asymptotically stable endemic equilibrium point when the reproduction number is greater than one. The simulation result shows the agreement with the analytical results.

Keywords: *Mathematical model, Corruption, Basic reproduction number, Stability*

1. Introduction

The World Bank (1997) defined corruption as the abuse of public office for private gain. In this sense, corrupt practices includes: bribery, extortion, fraud, embezzlement, nepotism, cronyism, appropriation of public institutions assets and properties for private use, and influence peddling. It is a symptom and result of institutional weakness, having negative effects on the economic performance of a country (Bardhan, 1997). Corruption is a complex and multifaceted phenomenon (Aidt, 2003) associated with all forms of human organization. As cited in Grass et al. (2008), the World Bank estimated the

annual cost of corruption over US \$80 billion worldwide which is more than the total of all economic assistance for development. Corruption distorts the rule of law and weakens the institutional foundations on which economic growth depends (World Bank, 2014) and thus identified among the greatest obstacles to economic and social development. Corruption degrades national security, economic prosperity and international reputation. It is the deep rooted cause of instability and conflict as witnessed in the present situation in Ethiopia.

Corruption could be characterized as

a “disease” inherent to public power and an indication of bad governance (Tiihonen, 2003). However, as reported in Blanchard et al. (2005), the features of corruption propagation differs from classical epidemic processes due to its dependence on the threshold value of the local transition probabilities and the mean field dependence of the corruption process. By this we mean that a non-corrupt individual gets infected with high probability if the number of corrupt individuals in the social neighborhood exceeds a certain threshold value whereas in the case of mean field dependence an individual can get corrupt because there is a high prevalence in the society even in the absence of corruption in the local neighborhood. In contrast to the high prevalence of corruption worldwide and large literature on political, social and economic aspects of corruption there is only a small number of attempts to model the dynamics of corruption in a mathematically quantified way. The first mathematical model dedicated to corrupt structures was appeared in (Rose-Ackerman, 1975) and become active research area since then. Several literatures showed that corruption smothering economic development across vast stretches of the globe, notably in Africa and Asia countries. For example, see in (Tanzi, 1998, Le Van & Maurel, 2006, Grass et al., 2008) and reference cited therein. It impacts negatively on a variety of economic and social well-being of the society and thus described as a worm within the body of the society and a cancer to economic, social, and

political development (Abdulrahman, 2014; Tanzi, 1998; Sayaji, 2013; Eguda et al., 2017). Corruption is an unavoidable part of human social interaction, prevalent in every society at any time since the very beginning of human history till today. It cannot be measured directly due to its elusive nature and the difficulty of separating the control of corruption from corruption itself.

The epidemiological corruption modeling approach was reported by many authors including (Starkermann, 1989; Le Van & Maurel, 2006; Becker et al., 2008; Brianzoni et al., 2011; Sayaji, 2013; Abdulrahman, 2014). The authors studied the effects of corruption on national development by modeling corruption as diseases. In particular, Hathroubi (2013) investigated an epidemiological corruption threshold based on the approximation of honest population by dividing the total population $N(t)$ into three compartments Susceptible $S(t)$, Corrupt $C(t)$, and Honest $H(t)$ individual without giving stability analysis of the equilibrium points. However, the author didn't considered well the case of losing immunity of jailed corrupts.

The objective of this work is to describe the transmission process of corruption, which can be defined generally as follows: when a reasonable amount of corrupted individuals are introduced into a susceptible population, the corruption is passed to other individuals through its modes of transmission, thus, spreading in the population. Corruption is assumed to be

a nonstandard epidemic process that rarely emerges out of nothing but is usually related to some already corrupt environment which may affect susceptible populations. Thus, in this article we will present a model for the spread of corruption in the spirit of epidemiology which describes the dynamical behavior of corruption, as a disease incorporating losing immunity of jailed and effort rate against corruption.

The article is structured as follows. In Section 2, we formulate our mathematical model for corruption transmission dynamics. We analyze the positivity and boundedness of solutions as well as the existence and local stability of the corruption-free and endemic equilibria in Section 3. Section 4 is devoted to numerical simulation. Finally, conclusions are given in Section 5.

2. Model formulation

In this section, we consider a SCJH (Susceptible-Corrupt-Jailed-Honest) type mathematical model for the dynamics of corruption. The total population $N(t)$ is divided into four compartments: Susceptible $S(t)$, Corrupt $C(t)$, in council through Jail $J(t)$ and Honesty $H(t)$ at time $t \geq 0$ as given in Abdulrahman (2014). In this paper, we meant by a susceptible individual that have never engaged in any corrupt practice or those jailed and then released after taking council. A corrupt is one that have engaged in a corruption practice and capable of influencing a susceptible individual to become corrupt. A jailed corrupt is one that is found to be guilty

corrupt by law and an honest individual is one that can never be corrupt under any situation.

We assume that there is a positive recruitment Λ into the susceptible class and a positive natural death rate μ for all time under the study. Susceptible individuals can become corrupt at rate $\frac{\beta C(t)}{\theta + C(t)}$ through contact with corrupt individuals which are depending on time. It can be increased or decreased based on the magnitude of θ and $C(t)$. The remaining model parameters are also defined as follows: τ is the effort rate against corruption, p is the corruption transmission probability per contact, θ is the maximum saturation constant of the corrupt population in the society, $\beta = p(1 - \tau)$ is the effective corruption contact rate, δ is the rate at which corrupt individuals are caught and imprisoned, $\frac{1}{\gamma}$ is the average period jailed individuals spent in prison, α is the proportion of individuals that leaves S , C and J to H compartment due to awareness created by anti-corruption agency through mass media, and counseling in jail respectively on the impact of corruption. It is assumed that the impact of awareness is same. The model also depends on the following assumptions: an individual can be corrupted only through contacts with corrupted individuals, a susceptible, corrupt, and jailed individual can be converted to honest due to awareness created by anti-corruption or counseling in jail. Compared to the work of Abdulrahman (2014), an individual who

lose immunity gained through council in jail do not directly joined corrupt class rather susceptible with the rate of $\gamma(1 - \alpha)$ because of human behavior. Further, our model takes into consideration the

impact due to awareness created by anti-corruption. The model proposed in Anthithan et al. (2018) do not consider the Jailed class which makes differ from our model.

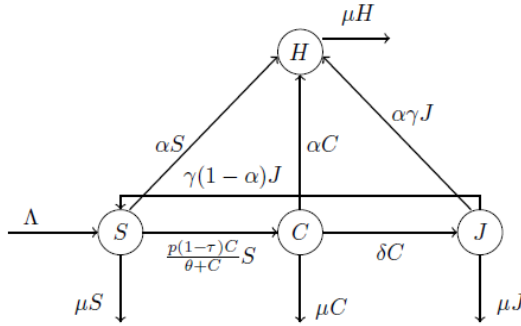


Figure 1. Schematic Diagram for Corruption Dynamics.

These assumptions are translated to the following mathematical model:

$$\begin{aligned} \frac{dS}{dt} &= A - \frac{\beta C}{\theta + C} S - (\alpha + \mu)S + \gamma(1 - \alpha)J \\ \frac{dC}{dt} &= A - \frac{\beta C}{\theta + C} S - (\delta + \alpha + \mu)C \\ \frac{dJ}{dt} &= \delta C - (\gamma + \mu)J \\ \frac{dH}{dt} &= \alpha(S + C + \gamma J) - \mu H. \end{aligned} \tag{2.1}$$

The model involves human population and hence all the parameters used are positive. Further, we assume that the initial conditions

$$\begin{aligned} S(0) \geq 0, C(0) \geq 0, J(0) \geq 0, H(0) \geq 0 \\ 0 \end{aligned} \tag{2.2}$$

of the governing equations are non-negative throughout this paper.

3. Model analysis

In this section, we study the solution of (2.1) in the epidemiologically feasible region.

$$\Omega = \left\{ (S, C, J, H) \in \mathbb{R}_+^4 : 0 \leq S(t) + C(t) + J(t) + H(t) \leq \frac{A}{\mu} \right\}.$$

3.1 Positivity and boundedness of solutions

Now we study the positivity and boundedness of the solution of the governing system (2.1).

Proposition 1. Assume that the initial conditions (2.2) hold in \mathbb{R}_+^4 . The the solutions $\{S(t), C(t), J(t), H(t)\}$ of the

model equations are also non-negative for all $t \geq 0$.

Proof. Assume that all the state variables are continuous. Then, from the system of equations (2.1) one can easily obtained that:

$$\frac{dS}{dt} \geq -\frac{\beta C}{\theta + C} S - (\alpha + \mu) S;$$

$$\frac{dC}{dt} \geq -(\delta + \alpha + \mu) C;$$

$$\frac{dJ}{dt} \geq -(\gamma + \mu) J;$$

$$\frac{dH}{dt} \geq -\mu H.$$

Solving these system of differential equations yields

$$S(t) \geq S(0)e^{\int (\frac{\beta C}{\theta + C} - (\alpha + \mu)) dt} \geq 0;$$

$$C(t) \geq C(0)e^{-(\delta + \alpha + \mu)t} \geq 0;$$

$$J(t) \geq J(0)e^{-(\gamma + \mu)t} \geq 0;$$

$$H(t) \geq H(0)e^{-(\mu)t} \geq 0.$$

Thus, we can conclude that all the solutions are non-negative in \mathbb{R}_+^4 for all $t \geq 0$.

The next proposition shows that it is sufficient to study the dynamics of corruption by model equations (2.1) in a region Ω .

Proposition 2. *Assume that all the initial conditions are non-negative in \mathbb{R}_+^4 for the system*

$$\Omega = \left\{ (S, C, J, H) \in \mathbb{R}_+^4 : 0 \leq S(t) + C(t) + J(t) + H(t) \leq \frac{\Lambda}{\mu} \right\}.$$

If $N(0) \leq \frac{\Lambda}{\mu}$, then the region Ω is positively invariant.

Proof. Note that all the state variables $S, C, J, H \in C(\mathbb{R}^+, \mathbb{R}^+)$ and the total population

$$N(t) = S(t) + C(t) + J(t) + H(t).$$

From this, we can easily deduce that

$$\frac{dN(t)}{dt} = \Lambda - \mu N(t).$$

Using the assumption that $N(0) \leq \frac{\Lambda}{\mu}$ and by integration we can have that $N(t) \leq \frac{\Lambda}{\mu}$ that is, $N(t)$ is for all bounded $t \geq 0$. This implies all the solutions of the model equation (2.1) with initial condition in Ω remains in Ω . This complete the proof.

Remark 1 In the region Ω , the proposed mathematical model is mathematically and epidemiologically well posed.

3.2 Stability Analysis

The corruption free equilibrium point (CFE) of the system (2.1) is given by

$$\bar{E} = (\bar{S}, \bar{C}, \bar{J}, \bar{H}) = \left(\frac{\Lambda}{\alpha + \mu}, 0, 0, \frac{\alpha \Lambda}{(\alpha + \mu)\mu} \right) \tag{3.1}$$

Following the approach of Abdulrahman (2014), Eguda (2017) & Anthithan.et al. (2018), we compute the basic reproduction number R_0 . In the present work, the basic reproductive number is the expected number of new corrupts from one corrupt individual in a fully susceptible population through contact period. Now we have the following proposition.

Proposition 3. *The basic reproduction number of the mathematical model (2.1) is given by*

$$R_o = \frac{\beta\Lambda}{(\alpha+\mu)(\delta+\alpha+\mu)\theta} \tag{3.2}$$

Proof. Let the matrix \mathbb{F} with the (i, j) entry denotes the number of new corrupts at stage j caused by contacts with corrupt individuals in stage i and the transition matrix V with (i, j) entry denotes the rate individuals in stage j progress to stage i . from model (2.1) we have $(S'(t), C'(t), J'(t), H'(t))^T = \mathbb{F} - V$ such that

$$\mathbb{F} = \begin{pmatrix} \frac{\beta C}{\alpha + C} S \\ 0 \end{pmatrix} \text{ and } = \begin{pmatrix} \alpha C \\ -\delta C + bJ \end{pmatrix}.$$

where $a = \delta + \alpha + \mu, b = \gamma + \mu$. The Jacobian matrices of \mathbb{F} and V at

equilibrium point are given by

$$F = \begin{pmatrix} \frac{\beta \tilde{S}}{\theta} & 0 \\ 0 & 0 \end{pmatrix} \text{ and } V = \begin{pmatrix} a & 0 \\ -\delta & b \end{pmatrix}.$$

Hence the basic reproduction number

$$R_o = \rho(FV^{-1}) = \frac{\beta\Lambda}{(\alpha + \mu)(\delta + \alpha + \mu)\theta}.$$

This concludes the proof.

Theorem 4. (Stability of the CFE) *The corruption-free equilibrium \tilde{E} of the dynamical system is*

- a) *Locally asymptotically stable if $\Lambda\beta < a(\alpha + \mu)\theta$ and unstable otherwise,*
- b) *Neutral if $\Lambda\beta = a(\alpha + \mu)\theta$.*

Proof. The characteristic polynomial of the corresponding linearized system of the governing equation is $p(\lambda) = \det((F - V) - \lambda I_4)$. That is,

$$\begin{vmatrix} -b - \lambda & \frac{\beta\Lambda}{(\alpha + \mu)\theta} & \gamma(1 - \alpha) & 0 \\ 0 & \frac{\beta\Lambda}{(\alpha + \mu)\theta} - a - \lambda & 0 & 0 \\ 0 & \delta & -b - \lambda & 0 \\ 0 & \alpha & \alpha\gamma & -\mu - \lambda \end{vmatrix} = 0$$

Evaluating the roots of this characteristic polynomial, we have

$$\lambda = -b, -b, \text{ or } \lambda = -\mu \text{ or } \lambda = \frac{\beta\Lambda}{(\alpha + \mu)\theta} - a.$$

As the model deals with human population, we assumed all the model parameters are non-negative.

Consequently, it is straightforward that the corruption-free equilibrium \tilde{E} is asymptotically stable for $\lambda = -b$ with

multiplicity two or $\lambda = -\mu$. On the other hand, the CFE \tilde{E} is locally asymptotically stable only if $\frac{\beta\Lambda}{(\alpha + \mu)\theta} - a < 0$ or equivalently,

$$\Lambda\beta < a(\alpha + \mu)\theta.$$

Trivially, a neutral case occurred if $\Lambda\beta = a(\alpha + \mu)\theta$ and it becomes inconclusive. With this we complete the proof of the theorem.

In the following theorem we shall investigate the existence of corruption endemic equilibrium point and its stability based on the computed reproduction number.

Theorem 5. *If $R_0 > 1$, then the*

i. Model (2.1) has a corruption endemic equilibrium point given by

$$\tilde{E} = (\tilde{S}, \tilde{C}, \tilde{J}, \tilde{H}) \tag{3.3}$$

where

$$\begin{aligned} \tilde{S} &= \frac{(\mu + \alpha + \delta)(\mu^2\theta + (\theta\alpha + \theta\gamma + \delta\theta + \Lambda)\mu + (\theta(\delta + 1)\alpha + \Lambda)\gamma)}{\mu^3 + (\gamma + 2\alpha + \beta + \delta)\mu^2 + (\alpha^2 + (\beta + 2\gamma + \delta)\alpha + (\beta + \delta)\gamma + \delta\beta)\mu + (\alpha + (\beta + 1)\delta + \beta)\alpha\gamma}; \\ \tilde{C} &= \frac{(\beta\Lambda - (\mu + \alpha)(\mu + \delta + \alpha)\theta)(\gamma + \mu)}{\mu^3 + (\gamma + 2\alpha + \beta + \delta)\mu^2 + (\alpha^2 + (\beta + 2\gamma + \delta)\alpha + (\beta + \delta)\gamma + \delta\beta)\mu + (\alpha + (\beta + 1)\delta + \beta)\alpha\gamma}; \\ \tilde{J} &= \frac{\delta(\beta\Lambda - (\mu + \alpha)(\mu + \delta + \alpha)\theta)}{\mu^3 + (\gamma + 2\alpha + \beta + \delta)\mu^2 + (\alpha^2 + (\beta + 2\gamma + \delta)\alpha + (\beta + \delta)\gamma + \delta\beta)\mu + (\alpha + (\beta + 1)\delta + \beta)\alpha\gamma}; \\ \tilde{H} &= \frac{((\Lambda - \theta\delta\gamma + \theta\delta)\mu^2 - ((\alpha\theta\delta - \Lambda + \theta\delta^2)\gamma - \theta\delta^2 + (-\Lambda - \theta\alpha)\delta - \Lambda(\alpha + \beta))\mu + \gamma((1 + \beta)\delta + \alpha + \beta)\Lambda)\alpha}{(\mu^3 + (\gamma + 2\alpha + \beta + \delta)\mu^2 + ((2\alpha + \beta + \delta)\gamma + (\delta + \alpha)(\alpha + \beta))\mu + \gamma\alpha((1 + \beta)\delta + \alpha + \beta))\mu}. \end{aligned}$$

ii. *Corruption endemic equilibrium is locally asymptotically stable.*

Proof. We set the left hand side of equation (2.1) equal to zero and solve it

$$\frac{\beta\tilde{C}}{\theta + \tilde{C}} = -\frac{(\gamma + \mu)((\mu + \alpha)(\mu + \delta + \alpha)\theta - \Lambda\beta)}{(\mu^2 + (\gamma + \alpha + \delta)\mu + \alpha\gamma(1 + \delta))\theta + \Lambda(\gamma + \mu)}.$$

This is equivalent to

$$\frac{\beta\tilde{C}}{\theta + \tilde{C}} = \frac{(\gamma + \mu)(\mu + \alpha)(\mu + \delta + \alpha)\theta\left(\frac{\beta\Lambda}{(\mu + \alpha)(\mu + \delta + \alpha)\theta} - 1\right)}{(\mu^2 + (\gamma + \alpha + \delta)\mu + \alpha\gamma(1 + \delta))\theta + \Lambda(\gamma + \mu)}.$$

By Proposition 3 $R_0 = \frac{\beta\Lambda}{(\alpha + \mu)(\delta + \alpha + \mu)\theta}$ and note that all parameters are positive. Hence, if $R_0 > 1$, then $\frac{\beta\tilde{C}}{\theta + \tilde{C}} > 0$. This

to obtain the equilibrium (3.3). From the effective corruption induced rate $\frac{\beta C(t)}{\theta + C(t)}$ at equilibrium we have

infer the existence of corruption in the specified population. With this we can conclude the existence of corruption endemic equilibrium point. To prove (ii)

we compute the Jacobian matrix of the system (2.1) at corruption endemic equilibrium \tilde{E} and then evaluate the eigenvalues as given in Theorem 3. Finally, we employ Routh-Hurwitz's criterion (Olsder & Woude, 2005) to show the corruption endemic equilibrium \tilde{E} of the system (2.1) is locally asymptotically stable for $R_0 > 1$. With this we conclude the proof of the theorem.

In the following section we detailed the solution of the proposed mathematical model through numerical simulation and compare its properties with analytical results.

4. Numerical simulation

This section presents and discusses numerical results for the system (2.1) for different values of parameters given in the model. The simulation process is carried out using Matlab and Maple for symbolic computations. We start by defining the value of each parameters used in the model developed. Then we illustrate the simulation results graphically.

4. 1 Parameter Estimations

The values of parameters given in Table 1 are borrowed from Abdulrahaman (2014) and TICPI (2017). The remaining parameters values are also carefully estimated and used in the simulation process. The assumed total population and recruitment rate is related by $\Lambda = \mu N$.

Table 1: Values of parameter used in the numerical simulation.

Parameter	Description	Value	Reference
Λ	Recruitment rate	μN	To be computed
p	Corruption transmission probability per contact	0.036	Abdulrahaman,S. (2014)
β	Effective corruption contact rate	0.0234	computed using TICPI, (2017)
τ	Effort rate against corruption	0.3500	TICPI, (2017) for Ethiopia
δ	Rate at which corrupt individuals are caught	0.0007	Abdulrahaman,S. (2014)
θ	Maximum saturation of corrupters population	100,000	Assumed
γ	Average rate of jailed individuals spent in prison	0.125	Abdulrahaman,S. (2014)
μ	Natural death rate	0.0160	WHO (2017) for Ethiopia
α	Proportion of individuals that joins H from each compartment due awareness	0.03	Assumed

4.2 Simulation results

The dynamics of corruption can be studied through numerical simulations by estimating the values of parameters in the model formulated with appropriate initial conditions. The population size is assumed to be $N = 700,000$ which is equivalent to the population size of medium city. If the total population $N = 700,000$, then $\Lambda = 11,182.108$. The corruption-free equilibrium state is given

$$\bar{E} = \left(\frac{\Lambda}{\alpha + \mu}, 0, 0, \frac{\alpha \Lambda}{(\alpha + \mu)\mu} \right) = (243224.4615, 0, 0, 456775.5389)$$

and the reproduction number $R_0 = 0.6097$. Figure 2 depicts that the susceptible population decreases asymptotically to the corruption-free equilibrium state while the honest population asymptotically increase to corruption-free equilibrium point.

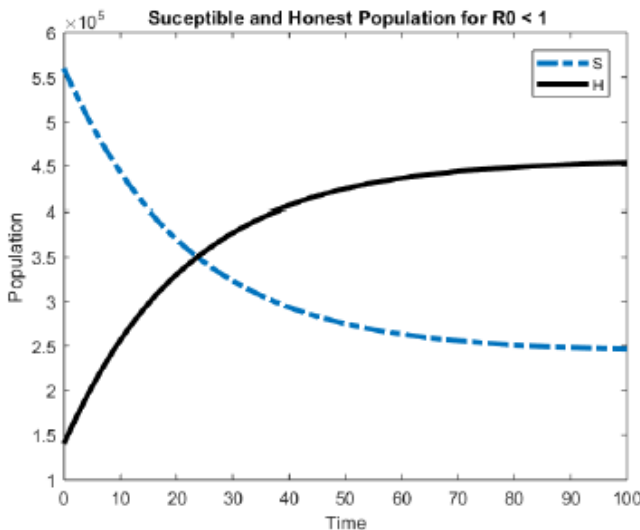


Figure 2. Susceptible and Honest population approaches asymptotically to corruption-free equilibrium state when $R_0 < 1$.

Thus we can observe that an agreement between the numerical simulation of the model (2.1) and the analysis of the local stability of the

corruption-free \bar{E} presented in Section (3).

The corruption endemic equilibrium point is

$$\tilde{E} = (228590.5911, 14602.76181, 72.50912434, 456734.1384)$$

For the parameter values given in Table 1. The basic reproduction number in this case is $R_0 = 1.22$ which is greater than one. Figure 3 clearly shows that all the state variables asymptotically approach

to the corruption endemic equilibrium point whenever $R_0 > 1$. Further, the corrupt population increases while the susceptible decrease to ward equilibrium point. This means, the susceptible

populations become influenced by corrupt individuals and the size of corrupt population raise in the society.

Also, the population of honest is significantly increasing due to awareness by anti-corruption and counseling in jail.

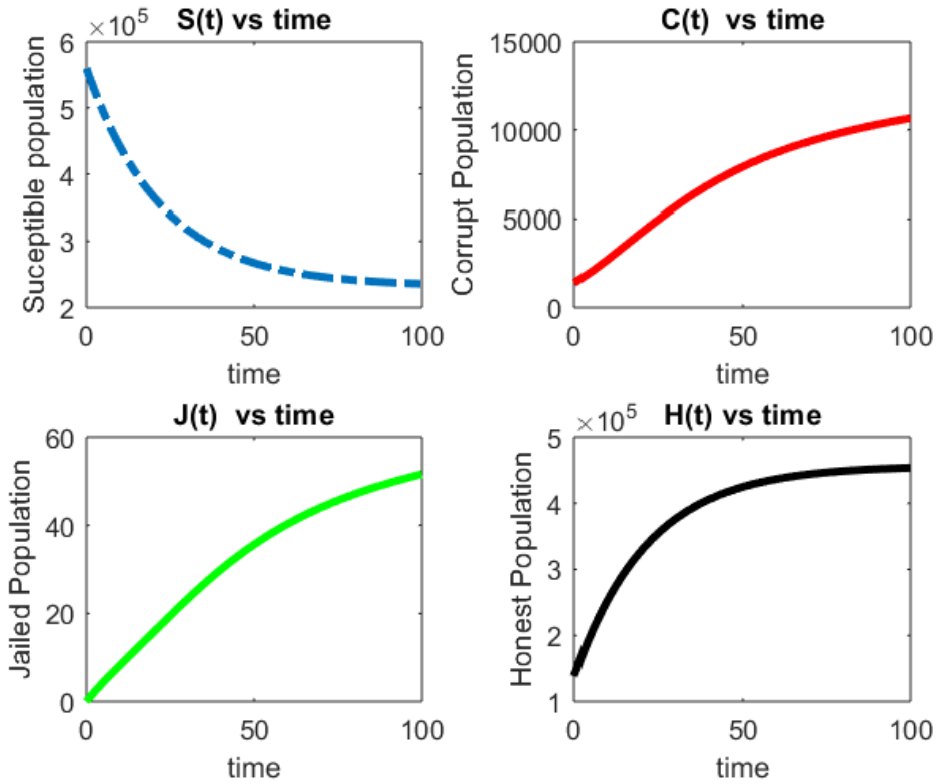


Figure 3. All the state variables approach asymptotically to corruption endemic equilibrium point when $R_0 > 1$.

As it can be seen in the Figure 3, the jailed population is slowly growing. This may be due to highly secretive nature of corruption and the corruptors are remaining in the community without arrested. For this reason, the number of susceptible individuals becomes decreases to the equilibrium which need optimal strategy to interfere. This part is left for future work.

By its nature corruption is secretive and its level of prevalence in the population is not uniform. Thus, the

dynamics of corruption for two countries namely, Ethiopia and New Zealand are comparatively studied based on Transparency International corruption perception index (TICPI, 2017) using the model developed. TICPI use a scale of 0 to 100 where 0 is highly corrupt and 100 is very clean. In this context, New Zealand rocked highest clean with score of 89 and Somalia is the most corrupt country with score 9. Ethiopia scored 35 and ranked 108 out of 180 countries. The penalties for corruption vary from country

to country. In some country it ranges from one year to life imprisonment and loss of public office in addition to repaying the amount involved or confiscating property which is gained as a result of corruption. The average period corrupt individuals spent in prison $1/\gamma$ is eight (8) years as cited in Abdulrahman (2014). The corruption endemic is almost

similar between 0 and 60 simulation time as illustrated in Figure 4. Beyond $t = 60$, the endemic is stronger in the case Ethiopia. This is due to the difference in corruption resistance in the two countries and thus, the proposed model well predicts the difference of corruption dynamics in the two countries.

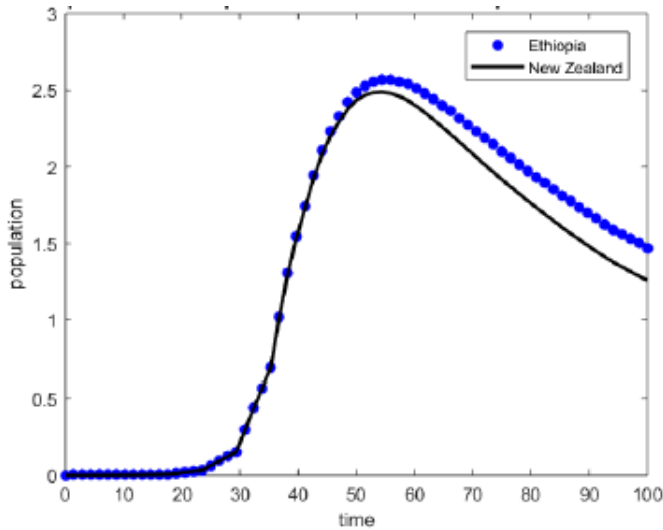


Figure 4. Comparison of corruption endemic in the case of Ethiopia and New Zealand based on TICPI (2017).

5. Conclusions

Corruption challenges good governance and become a hinder for investment. In this paper, we developed and analyzed a mathematical model to study the dynamical nature of corruption based on the governing mathematical model with constant recruitment rate from the total population. The domain in which the model is mathematically and epidemiologically well posed was determined. Then the next generation matrix was used to derive the basic

reproduction number. The corruption-free equilibrium of the model was proved to be locally asymptotically stable whenever reproduction number is less than one. It is also showed that the corruption epidemic equilibrium is locally asymptotically stable provided that the basic reproduction number is greater than one. Finally, the analytical result was verified using numerical simulations. Designing optimal control strategy is left for future work.

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