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Research Paper

Generation of Kifilideen's Generalized Matrix Progression Sequence of Infinite Term

Kifilideen L. Osanyinpeju*

Department of Agricultural and Bio-Resources Engineering (ABE), COLENG, Federal University of Agriculture, Alabata, Abeokuta, Ogun State, Nigeria

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Abstract

Considering a cluster of different hierarchical order with various barrier or cadre levels and steps within levels, there is no availability of a structural framework to help exclusively analyze, formulate, identify, differentiate, and design standardized provisional values to cluster members at various barrier levels and steps within levels. This study designs stepwise analysis, generation, and applications of Kifilideen's Matrix Structural Framework for an infinite term of increasing members of successive levels with the first level having one member of Kifilideen's Generalized Matrix Progression Sequence. The values of members of the cluster were designed and developed for Kifilideen's Generalized Matrix Progression Sequence of the infinite term of increasing members of successive levels with the first level having one member and Kifilideen's Structural Framework was generated for the clusters of such sequence. This Kifilideen's Matrix Structural Framework also helps to generate Kifilideen's formulas to identify members and assign values and grades of values to each member within and across levels of Kifilideen's Matrix Structural Framework. Applications of Kifilideen's formulas established for Kifilideen's Generalized Matrix Progression Sequence of the infinite term was carried out. Kifilideen's Matrix Structural Framework generated some help to exclusively differentiate varying members in the clusters into various levels and steps within levels.

1. Introduction

Mathematics is the backbone of a nation and scientific world which enables scientists to have accurate and precise predictions of future events in the area of forecasting (Kolawole, 2004; Kolawole and Ojo, 2019; Osanyinpeju, 2021). Mathematics develops thinking, reasoning, and problem-solving skills that would serve as a weapon for a person to overcome any encounter in the real world (Bonotto and Santo, 2015; Ozdemir and Celik, 2021; Osanyinpeju, 2022). To excel in mathematics, mathematical thinking and reasoning must come into play in the establishment of fact (Osanyinpeju et al., 2019; Ozdemir and Celik, 2020). Considering a cluster of different hierarchical order with various barrier levels; designing a structural framework would help to effectively and exclusively identify, differentiate, analyze, formulate, and design provisional values to cluster members at various barrier levels and steps within the levels. There is no – availability of a structural flow for the organization of members of clusters into clustered frameworks of different levels and steps. Matrix progression sequence into structural framework has practical application in the area of hierarchical clusters, competition for resources, and sharing formula among members of cluster and food chain. The educational sector is in a hierarchical order which can be formulated into a structural framework (Moja, 2000; Ikechukwu,

*Corresponding author, e-mail: amkifilideenosanyinpeju@gmail.com

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2015). The educational sector is divided into levels which are nursery, primary, secondary, and tertiary levels which represent the levels of a structural framework (Amaghionyeodiwe and Osinubi, 2006; Ezeanochie and Alamgir, 2021). At each level, there are categories of steps that represent the steps in the structural framework. For example, the primary level has primary 1 to 6 which serves as the steps; the secondary school level has JS 1 – 3 and SS 1 – 3 while the tertiary level has 100 – 500 levels (Ndidi, 2013; Nakpodia, 2020; Kinika-Nsirim and Okeah, 2021).

The nursery level has the largest number of students which in turn requires the largest number of teachers/members/staff in the clusters of the educational sectors. The rewards given to members/staff/teachers in the highest step in the nursery level is the smallest reward compared to the highest steps in other levels (primary, secondary, and tertiary levels) of the educational sector. As we move down the steps in the nursery levels, the reward decreases. What contributes to the lowest reward in the nursery level compared to other levels in the educational sector are low skill, low qualification, simplicity of job, low demand of effort, low task and effort, low input, and low level of expertise. There is a hierarchy within the nursery level in which staff/members/teachers are in various steps depending on their qualifications, experience, and number of years spent in the organization. The most senior staff/officer in the nursery level has the highest reward or pay or remuneration and the reward or pay or remuneration decreases as the hierarchy drops at the nursery level in the structural framework. To migrate from the nursery level to another level, there is a need to meet up with the criteria required at the higher level. Also, within a nursery level or other levels, members can migrate from one step to another which can come as a result of the addition of more skill, experience, exposure in the field, and more years spent in the level.

As we migrate to a higher level in the structural framework, the number of steps and the number of members reduce. The highest step in the primary level receives more rewards than the highest step in the nursery level. Within the primary level, as we move down, the level degree of steps drops so, the reward decreases. Within every level in the educational sector, hierarchical order is present. Furthermore, at the secondary level, the number of students is further reduced which may come as a result of a loss of interest on the part of the student, insufficient funds to further the education, no availability of sponsor, having low potential to cope, low performance and more effectiveness to trade. The lesser number of students in the secondary school results in a lesser number of teachers; although the reward is more for each teacher/member/staff at this level due to the complexity of the task they undergo, more skill, and more expertise. Migrating to the tertiary level, the number of students drops more alongside the number of teachers/staff. The drop in the number of students is a result of some students not meeting the prerequisite to be students in the tertiary; and the tertiary level is more demanding, making only the strong ones among be able to proceed. Also moving to a higher level requires more funds which some students may not be able to meet up. Each teacher/member/staff at this level gets more reward than the lower levels (nursery, primary, and secondary levels) because teacher/member/staff at this level uses sophisticated and complex tools that require more skill, advancement, expertise, experience, input, effort and relative complexity. For each of the levels, there is a hierarchical order within their level where each member within a level is placed in step. For example, at the secondary school level; staff handling the senior class would be placed at a higher step and be rewarded more than the staff in the junior class. Placing the member of the cluster of the educational sector in a structural framework would help to formulate a sharing formula of reward to the member of the cluster of the sector.

Furthermore, in the secondary school educational sector, a cluster of members is formed in a hierarchical order which is grouped into levels which are subject teachers, head of department HoD, vice principal, and principal (Bello et al., 2016; Munje et al., 2020; Irvine, 2022). The order of increase in levels spans from subject teachers, head of department HoD, vice principal and principal (Maja, 2016; Osuji and Etuketu, 2019; Abdullah and Salihu, 2020). As we migrate from one level to higher levels the number of members decreases. The class teacher has the largest number of members but each member in that level is less skilled and less rewarded compared to the other levels such as the head of department (HoD), vice principal and principal. At the head of a department level, more skill, input, and diversity of knowledge and experience which bring more reward are required, the same trend is applied to the vice principal and principal. The principal has the lowest number of members but the most skilled, and highly experienced in the field, and has the highest complexity of tasks. Within each level, there is a presence of steps. Each member of a particular level would pass through steps to get to the peak step of such a level. To migrate from one level to another, members have to upgrade themselves to fit into the next level which results in more rewards for such members (Arikewuyo, 2009). The cluster of members in the secondary school sector can be formulated into the structural framework.

A Kifilideen's Clustered Framework presented in this paper is considered a set of entities with related attributes but unique in their representation of identity thus, bringing about different levels and steps within the levels. More so, levels are considered as columns while steps are considered as rows in Kifilideen's Matrix Structural Framework of the members of the clusters. Image an array of related sequential Kifilideen's Generalized Matrix Progression of infinite term for a cluster, to expand the chain as the trend advances say in order of Table 1, where each term is a function defined as Tn for {n=1,2,3,4,…}.

The infinite term of Kifilideen's Generalized Matrix Progression Sequence invented in this paper has one member in the first level with increasing members for successive levels, unlike the finite terms of Kifilideen's General Matrix Progression counterpart which has (h+1) members in the first level. The members of the infinite term of Kifilideen's Generalized Matrix Progression Sequence are arranged in such a way that the number of members increases by a magnitude of one from one successive level to another and then to an infinite number of members at the infinity level.

The Kifilideen's mathematical formulas inaugurated for the infinite term of Kifilideen's Generalized Matrix Progression Sequence are simplified and appear simple as a result of the one member in the first level. Meanwhile, Kifilideen's formulas generated for the finite terms counterpart of Kifilideen's Generalized Matrix Progression Sequence are more complex due to the presence of (h+1) members in the first level. For both finite and infinite terms of the generalization of Kifilideen's Generalized Matrix Progression Sequence, the difference in the number of members of two successive levels is one in Kifilideen's Matrix Structural Framework. A level stops absorbing members in Kifilideen's Matrix Structural Framework when the coefficient of migration level value, k, and the coefficient of migration step value, i the same for the infinite term of Kifilideen's Generalized Matrix Progression Sequence.

Migration level value, k is the difference between the values of the terms in the first positions of two successive levels in Kifilideen's Matrix Structural Framework of Kifilideen's Generalized Matrix Progression Sequence of infinite and finite terms while Kifilideen step value, i is the difference between two successive steps within a level of the Kifilideen's Matrix Structural Framework of Kifilideen's Generalized Matrix Progression Sequence of infinite and finite terms (Osanyinpeju, 2020a; Osanyinpeju, 2020b). This study designs stepwise analysis, generation, and applications of Kifilideen's Matrix Structural Framework for an infinite term of increasing members of successive levels with the first level having one member of Kifilideen's Generalized Matrix Progression Sequence.

Table 1. The Kifilideen's Matrix Structural Framework for an infinite term

	$Level\ 1$	$Level\ 2$	$Level\ 3$	$Level\ 4$	$Level\ 5$	...
$Step\ 1$	T_1					
$Step\ 2$		T_2				
$Step\ 3$		T_3	T_4			
$Step\ 4$			T_5	T_7		
$Step\ 5$			T_6	T_8	T_{11}	
$Step\ 6$				T_9	T_{12}	.
$Step\ 7$				T_{10}	T_{13}	..
$Step\ 8$					T_{14}	...
$Step\ 9$					T_{15}	...
$Step\ 10$						...
$Step\ 11$						...

2. Materials and Methods

The methodology used in proving all the Kifilideen's mathematical formulation of the components of the generation of the infinite term of increasing members of successive levels with the first level having one member of Kifilideen's Generalized Matrix Progression Sequence is, proved by mathematical induction in this paper.

2.1. System of Kifilideen Generalized Matrix Progression Sequence of Infinite Term

The system of progression of Kifilideen's Generalized Matrix Sequence of the infinite term of increasing members of successive levels with the first level having one member is generated as

$$k(0)+i(0)+f; k(1)+i(0)+f,\ \ k(1)+i(1)+f;\ k(2)+i(0)+f,\ \ k(2)+i(1)+f, k(2)+i(2)+f;$$
$$k(3)+i(0)+f, k(3)+i(1)+f, k(3)+i(2)+f, k(3)+i(3)+f;\ k(4)+i(0)+f, k(4)+i(1)+f,$$
$$k(4)+i(2)+f, k(4)+i(3)+f, k(4)+i(4)+f;\ k(5)+i(0)+f, k(5)+i(1)+f, k(5)+i(2)+f,$$
$$k(5)+i(3)+f, k(5)+i(4)+f, k(5)+i(5)+f;\ \ k(6)+i(0)+f, k(6)+i(1)+f, k(6)+i(2)+f,$$
$$k(6)+i(3)+f, k(6)+i(4)+f, k(6)+i(5)+f, k(6)+i(6)+f;\ \ k(7)+i(0)+f, k(7)+i(1)+f,$$
$$k(7)+i(2)+f, k(7)+i(3)+f, k(7)+i(4)+f, k(7)+i(5)+f, k(7)+i(6)+f, k(7)+i(7)+f; \ldots.$$

The Kifilideen's Generalized Matrix Sequence of the infinite term of increasing members of successive levels with the first level having one member has endless terms. The progression of the sequence begins without end. The placement of the progression of each term of the infinite term of Kifilideen's Generalized Matrix Sequence in standardized order in Kifilideen's Matrix Structural Framework is displayed in Table 2. From the Table 2; Level 1, 2, 3,4, 5, … has $1, 2, 3, 4, 5, \ldots$ member (s) respectively. The coefficients of k and i are the same for the last member in each level.

The coefficient of k for member (s) in levels 1, 2, 3,4, 5, … are 0, 1, 2, 3, 4, … respectively. The coefficient of i is increasing in the level from 0 to one magnitude less than the value of the level in the analyzed level. For any particular sequence, the values of k, i and f have fixed values. k, i and f are the migration level value, migration step value and first term respectively. From the analysis of Table 2, generally steps 1 and 2; 3 and 4; 5 and 6; 7 and 8;... have 1; 2; 3; 4;... member (s) respectively. The value of the coefficient of k is fixed for any particular level and step but varies from one level and step to another level and step. From one level to another successive level the number of members increases by one. The first level has one member.

Table 2: The placements of infinite terms in Kifilideen's Matrix Structural Framework.

	l_1	l_2	l_3	l_4	l_5	...
s_1	$k(0)+i(0)+f$					
s_2		$k(1)+i(0)+f$				
s_3		$k(1)+i(1)+f$	$k(2)+i(0)+f$			
s_4			$k(2)+i(1)+f$	$k(3)+i(0)+f$		
s_5			$k(2)+i(2)+f$	$k(3)+i(1)+f$	$k(4)+i(0)+f$	
s_6				$k(3)+i(2)+f$	$k(4)+i(1)+f$	.
s_7				$k(3)+i(3)+f$	$k(4)+i(2)+f$	..
s_8					$k(4)+i(3)+f$	…
s_9					$k(4)+i(4)+f$	…
s_{10}						…
s_{11}						…

2.2. Kifilideen's Term Mathematical Formula of Infinite Term

The stepwise mathematical induction of the generation of the Kifilideen's term mathematical formula of the infinite term of the Kifilideen's Generalized Matrix Progression Sequence is analyzed as follows:

Level 1, $l = 1$;
$$T_1 = k(0) + i(0) + f \quad (1)$$
$$T_1 = k(1-1) + i(1-1) + f \quad (2)$$
Level 2, $l = 2$;
$$T_2 = k(1) + i(0) + f \quad (3)$$
$$T_2 = k(2-1) + i(2-2) + f \quad (4)$$
$$T_3 = k(1) + i(1) + f \quad (5)$$
$$T_3 = k(2-1) + i(3-2) + f \quad (6)$$
Level 3, $l = 3$;
$$T_4 = k(2) + i(0) + f \quad (7)$$
$$T_4 = k(3-1) + i(4-4) + f \quad (8)$$
$$T_5 = k(2) + i(1) + f \quad (9)$$
$$T_5 = k(3-1) + i(5-4) + f \quad (10)$$
$$T_6 = k(2) + i(2) + f \quad (11)$$
$$T_6 = k(3-1) + i(6-4) + f \quad (12)$$
Level 4, $l = 4$;
$$T_7 = k(3) + i(0) + f \quad (13)$$
$$T_7 = k(4-1) + i(7-7) + f \quad (14)$$
$$T_8 = k(3) + i(1) + f \quad (15)$$
$$T_8 = k(4-1) + i(8-7) + f \quad (16)$$
$$T_9 = k(3) + i(2) + f \quad (17)$$
$$T_9 = k(4-1) + i(9-7) + f \quad (18)$$
$$T_{10} = k(3) + i(3) + f \quad (19)$$
$$T_{10} = k(4-1) + i(10-7) + f \quad (20)$$
Level 5, $l = 5$;
$$T_{11} = k(4) + i(0) + f \quad (21)$$
$$T_{11} = k(5-1) + i(11-11) + f \quad (22)$$
$$T_{12} = k(4) + i(1) + f \quad (23)$$
$$T_{12} = k(5-1) + i(12-11) + f \quad (24)$$
$$T_{13} = k(4) + i(2) + f \quad (25)$$
$$T_{13} = k(5-1) + i(13-11) + f \quad (26)$$
$$T_{14} = k(4) + i(3) + f \quad (27)$$
$$T_{14} = k(5-1) + i(14-11) + f \quad (28)$$
$$T_{15} = k(4) + i(4) + f \quad (29)$$
$$T_{15} = k(5-1) + i(15-11) + f \quad (30)$$
.
.
.
Level l, $l = l$;
$$T_n = k(a) + i(s) + f \quad (31)$$
$$T_n = k(l-1) + i(n-m) + f \quad (32)$$
.
.
.

Generally, from the stepwise mathematical induction of the Kifilideen's term mathematical formula of the infinite term of Kifilideen's Generalized Matrix Progression Sequence increasing members for successive levels with the first level having one member is achieved as:
$$T_n = k(l-1) + i(s) + f \quad (33)$$
$$T_n = k(a) + i(n-m) + f \quad (34)$$
Comparing (33) and (34),
$$a = l - 1 \text{ and } s = n - m \quad (35)$$
Where T_n is the value of the n^{th} the term, f is the first term, k is the migration level value, i is the migration step value, n is the number of terms, a is the migration level factor, m is the migration step factor, l is the level value of the term and s is the migration term step difference factor.

Table 3 presents the value of $l, a,$ and m for each level of the Kifilideen's Matrix Structural Framework of the infinite term of the Kifilideen's Generalized Matrix Progression Sequence of increasing members of successive levels with the first level having one member. From the Table 3, when the values of l (level) are $1, 2, 3, 4, 5, 6, \ldots, l, \ldots, \ldots$ the values of a are $0, 1, 2, 3, 4, 5, \ldots, a, \ldots, \ldots$ respectively while the values of m are $(1) \rightarrow 1, (1+1) \rightarrow 2, (1+1+2) \rightarrow 4, (1+1+2+3) \rightarrow 7, (1+1+2+3+4) \rightarrow 11, (1+1+2+3+4+5) \rightarrow 16, \ldots, m, \ldots, \ldots$ respectively.

So, generally, for an infinite term of the Kifilideen's Generalized Matrix Progression Sequence of increasing members for successive levels with the first level having one member, we have:

For $a = l - 1, m = 1 + 1 + 2 + 3 + 4 + 5 + \cdots$ (36)

Let $m = 1 + \beta$, where $\beta = 1 + 2 + 3 + 4 + 5 + \cdots$ (37)

The series of β is an arithmetic progression series. From Table 3, omitting the first term of the series of m in the Table 3, the number of terms of β is equivalent to the value of a. Using the summation of the arithmetic progression formula showcased by Stroud and Booth (2007), Oluwasanmi (2011); we have:

$$S_z = \frac{z}{2}(2w + (z - 1)d) \quad (38)$$

Where S_z is the sum of the series, β, z is the number of terms of the series, β, w is the first term, and d is the common difference between two successive terms of the series, β.

From (37), $S_z = \beta$, $w = 1, d = T_2 - T_1 = 2 - 1 = 2, z = a$ = migration level factor (39)

$z = a$, Since it has been deduced that the number of terms of the series of β is equivalent to the value of a when the first term of the series of m is omitted in Table 3.

$$S_z = \beta = \frac{a}{2}(2 \times 1 + (a - 1) \times 1) \quad (40)$$

$$\beta = \frac{a}{2}(2 + a - 1) \quad (41)$$

$$\beta = \frac{a(a+1)}{2} \quad (42)$$

Put (42) in (37), we have:

$$m = 1 + \beta = 1 + \frac{a(a+1)}{2} \quad (43)$$

$$m = \frac{2+a(a+1)}{2} \quad (44)$$

Table 3. The value of l, a and m for each level of the Kifilideen's Matrix Structural Framework of the infinite term.

l	a	m
1	0	$1 = 1$
2	1	$1 + 1 = 2$
3	2	$1 + 1 + 2 = 4$
4	3	$1 + 1 + 2 + 3 = 7$
5	4	$1 + 1 + 2 + 3 + 4 = 11$
.	.	.
.	.	.
l	$l - 1$	$1 + 1 + 2 + 3 + 4 + 5 + 6 + \cdots + (l - 1) = 1 + 1 + 2 + 3 + 4 + 5 + 6 + \cdots + a$
.	.	.
.	.	.
.	.	.

$$m = \frac{a^2+a+2}{2} \quad (45)$$

$$a = l - 1 \quad (46)$$

Put (46) in (45), we have:

$$m = \frac{(l-1)^2+(l-1)+2}{2} \quad (47)$$

$$m = \frac{l^2-2l+1+l-1+2}{2} \quad (48)$$

$$m = \frac{l^2-l+2}{2} \quad (49)$$

In all, the Kifilideen's term mathematical formula of the infinite term of Kifilideen's Generalized Matrix Progression Sequence of increasing members of successive levels having one member in the first level is obtained as:

$$T_n = k(a) + i(n - m) + f \quad (50)$$

$$a = l - 1, \quad m = \frac{a^2+a+2}{2}, \; m = \frac{l^2-l+2}{2} \quad (51)$$

Where T_n is the value of the n[th] term, f is the first term, k is the migration level value, i is the migration step value, n is the number of terms, a is the migration level factor, m is migration step factor, l is the level value of the term and s is the migration term step difference factor.

2.3. Mathematical Formulation of the Migration Level Factor, a of Infinite Term

Table 4 layouts the placement of the terms of Kifilideen's Generalized Matrix Progression Sequence of increasing members of successive levels having one member in the first level. The mathematical formulation of the migration level factor, a of the infinite term of the Kifilideen's Generalized Matrix Progression Sequence of increasing members of successive levels having one member in the first level is demonstrated as follows:

From Table 4, level 1 contains T_1 having $a = 0$; level 2 contains T_2, T_3 all having $a = 1$; level 3 contains T_4, T_5, T_6 all having $a = 2$; level 4 contains T_7, T_8, T_9, T_{10} all having $a = 3$; level 5 contains $T_{11}, T_{12}, T_{13}, T_{14}, T_{15}$ all having $a = 4$; ... , ... , ...

Table 3 displays the relationship of the migration level factor, a and the number of term of the first member of each level. Taking into consideration the term of the first member of each level, we have:

Level 1, $a = 0$, $n = 1 = 1$ (52)

Level 2, $a = 1$, $n = 1 + 1 = 2$ (53)

Level 3, $a = 2$, $n = 1 + 1 + 2 = 4$ (54)

Level 4, $a = 3$, $n = 1 + 1 + 2 + 3 = 7$ (55)

Level 5, $a = 4$, $n = 1 + 1 + 2 + 3 + 4 = 11$ (56)

.
.
.

Level l, $a = l - 1$, $n = 1 + 1 + 2 + 3 + 4 + 5 + 6 + \cdots + a$ (57)

.
.
.

Let $n = 1 + \varphi$ in the series of n in (57)

$$\varphi = 1 + 2 + 3 + 4 + 5 + \cdots \quad (58)$$

The series of φ is an arithmetic progression series. From the summation of the arithmetic progression formula presented by Ilori *et al.* (2000), and Nwabuwanne (2001); we have:

$$S_q = \frac{q}{2}(2w + (q - 1)d) \quad (59)$$

Where S_q is a sum of the series, φ, q is the number of terms of the series,φ, w is the first term, and d is the common difference between two successive terms of the series, φ. From Table 3, omitting the first term of the series n, the number of terms of φ is equivalent to the value of a.

From (58), $S_q = \varphi, q = a$ =migration level factor,

$$w = 1, d = T_2 - T_1 = 2 - 1 = 1 \quad (60)$$

$$S_q = \varphi = \frac{a}{2}(2 \times 1 + (a - 1) \times 1) \quad (61)$$

$$\varphi = \frac{a}{2}(2 + a - 1) \quad (62)$$

$$\varphi = \frac{a(a+1)}{2} \quad (63)$$

Table 4: The placement of the infinite term of the Kifilideen's Generalized Matrix Progression Sequence

	l_1	l_2	l_3	l_4	l_5	...
s_1	T_1					
s_2		T_2				
s_3		T_3	T_4			
s_4			T_5	T_7		
s_5			T_6	T_8	T_{11}	
s_6				T_9	T_{12}	
s_7				T_{10}	T_{13}	.
s_8					T_{14}	..
s_9					T_{15}	...
s_{10}						...
r_{11}						...
r_{12}						...

Put (63) in (58), we have:

$$n = 1 + \varphi = 1 + \frac{a(a+1)}{2} \quad (64)$$

$$n = \frac{a^2+a+2}{2} \quad (65)$$

$$a^2 + a + 2 = 2n \quad (66)$$

$$a^2 + a + 2 - 2n = 0 \quad (67)$$

Using the quadratic formula presented by Adu (2004), Asuquo *et al.* (2007); we have:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad (68)$$

$$a = 1, b = 1, c = 2 - 2n \text{ and } x = a \quad (69)$$

$$a = \frac{-1 \pm \sqrt{(1)^2 - 4 \times 1 \times (2 - 2n)}}{2 \times 1} \quad (70)$$

$$a = \frac{-1 \pm \sqrt{1 - 8 + 8n}}{2 \times 1} \quad (71)$$

$$a = \frac{-1 \pm \sqrt{8n - 7}}{2} \quad (72)$$

Since a is positive, therefore:

$$a = \frac{-1 + \sqrt{8n - 7}}{2} \quad (73)$$

From Table 4, level 1 contains T_1 having $a = 0$; level 2 contains T_2, T_3 all having $a = 1$; level 3 contains T_4, T_5, T_6 all having $a = 2$; level 4 contains T_7, T_8, T_9, T_{10} all having $a = 3$; level 5 contains $T_{11}, T_{12}, T_{13}, T_{14}, T_{15}$ all having $a = 4$; ... , ... , ...

The value of a for the first term of each level in Kifilideen's Matrix Structural Framework is the same as the other terms in that level. This indicates that the value of a is the same for all terms in the same level. So, (85) can be used to obtain the value of the migration level factor, a for any term of the infinite term of Kifilideen's Generalized Matrix Progression Sequence for increasing members of successive levels and having one member in the first level.

Generally, the Kifilideen's term mathematical formula for an infinite term of Kifilideen's Generalized Matrix Progression Sequence of increasing members of successive levels and having one member in the first level is achieved as:

$$T_n = k(a) + i(n - m) + f \quad (74)$$

$$a = \text{Migration level factor} = \frac{-1+\sqrt{8n-7}}{2} \quad (75)$$

$$m = \text{Migration step factor} = m = \frac{a^2+a+2}{2} = \frac{l^2-l+2}{2} \quad (76)$$

2.4. Kifilideen's Level Mathematical Formula for the Infinite Term

The generation of the Kifilideen's level formula for the infinite term of the Kifilideen's Generalized Matrix Progression Sequence of increasing members of successive levels and having one member in first level is generated as:

From (51), we have:

$$l = a + 1 \quad (77)$$

Put (73) into (77), we have:

$$l = \frac{-1+\sqrt{8n-7}}{2} + 1 \quad (78)$$

$$l = \frac{1+\sqrt{8n-7}}{2} \quad (79)$$

The Kifilideen's level mathematical formula for the infinite term of the Kifilideen's Generalized Matrix Progression Sequence is attained as:

$$l = \frac{1+\sqrt{8n-7}}{2} \quad (80)$$

Where $n$the number of the term of the Kifilideen is's Generalized Matrix Progression Sequence and l is the level value of the term in the Kifilideen's Matrix Structural Framework of the Kifilideen's Generalized Matrix Progression Sequence.

2.5. Kifilideen's Position Mathematical Formula of the Infinite Term

Table 5 displays the position of each term of the infinite term of Kifilideen's Generalized Matrix Progression Sequence of increasing members of successive levels and one member in the first level in Kifilideen's Matrix Structural Framework. In level 1: T_1 is in position 1; in level 2: T_2, T_3 are in positions 1 and 2 respectively; in level 3: T_4, T_5, T_6 are in position 1, 2 and 3 respectively; in level 4: T_7, T_8, T_9, T_{10} are in position 1, 2, 3 and 4 respectively; in level 5: $T_{11}, T_{12}, T_{13}, T_{14}, T_{15}$ are in position 1, 2, 3, 4 and 5 respectively; …, …, …,

The stepwise analysis of the mathematical induction of the Kifilideen's position mathematical formula of the infinite term of the Kifilideen's Generalized Matrix Progression Sequence of increasing members of successive levels and one member in the first level is illustrated as follows:

Level **1**; position $= p =$ **1**;
$$T_1 = k(0) + i(0) + f \quad (81)$$
$$T_1 = k(\mathbf{1} - 1) + i(\mathbf{1} - 1) + f \quad (82)$$

Level **2**; position $= p =$ **1**;
$$T_2 = k(1) + i(0) + f \quad (83)$$
$$T_2 = k(\mathbf{2} - 1) + i(\mathbf{1} - 1) + f \quad (84)$$

Level **2**; position $= p =$ **2**;
$$T_3 = k(1) + i(1) + f \quad (85)$$
$$T_3 = k(\mathbf{2} - 1) + i(\mathbf{2} - 1) + f \quad (86)$$

Level **3**; position $= p =$ **1**;
$$T_4 = k(2) + i(0) + f \quad (87)$$
$$T_4 = k(\mathbf{3} - 1) + i(\mathbf{1} - 1) + f \quad (88)$$

Level **3**; position $= p =$ **2**;
$$T_5 = k(2) + i(1) + f \quad (89)$$
$$T_5 = k(\mathbf{3} - 1) + i(\mathbf{2} - 1) + f \quad (90)$$

Level **3**; position $= p =$ **3**;
$$T_6 = k(2) + i(2) + f \quad (91)$$
$$T_6 = k(\mathbf{3} - 1) + i(\mathbf{3} - 1) + f \quad (92)$$

Level **4**; position $= p =$ **1**;
$$T_7 = k(1) + i(0) + f \quad (93)$$
$$T_7 = k(\mathbf{2} - 1) + i(\mathbf{1} - 1) + f \quad (94)$$

Level **4**; position $= p =$ **2**;
$$T_8 = k(1) + i(1) + f \quad (95)$$
$$T_8 = k(\mathbf{2} - 1) + i(\mathbf{2} - 1) + f \quad (96)$$

Level **4**; position $= p =$ **3**;
$$T_9 = k(1) + i(2) + f \quad (97)$$
$$T_9 = k(\mathbf{2} - 1) + i(\mathbf{3} - 1) + f \quad (98)$$

Level **4**; position $= p =$ **4**;
$$T_{10} = k(1) + i(3) + f \quad (99)$$
$$T_{10} = k(\mathbf{2} - 1) + i(\mathbf{4} - 1) + f \quad (100)$$

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Level; position $= p = \boldsymbol{p}$;
$$T_n = k(\boldsymbol{l} - 1) + i(\boldsymbol{p} - 1) + f \quad (101)$$

Table 5: The position of each term of the infinite term in Kifilideen's Matrix Structural Framework.

	l_1	l_2	l_3	l_4	l_5	...
s_1	$p_1 \to T_1$					
s_2		$p_1 \to T_2$				
s_3		$p_2 \to T_3$	$p_1 \to T_4$			
s_4			$p_2 \to T_5$	$p_1 \to T_7$		
s_5			$p_3 \to T_6$	$p_2 \to T_8$	$p_1 \to T_{11}$	
s_6				$p_3 \to T_9$	$p_2 \to T_{12}$	
s_7				$p_4 \to T_{10}$	$p_3 \to T_{13}$	.
s_8					$p_4 \to T_{14}$	..
s_9					$p_5 \to T_{15}$	...
s_{10}						...
s_{11}						...
s_{12}						...

Generally, the Kifilideen's position mathematical formula for Kifilideen's Generalized Matrix Progression Sequence is generated as:

$$T_n = k(\boldsymbol{l} - 1) + i(\boldsymbol{p} - 1) + f \quad (102)$$

Comparing (74) and (102), we have:

$$n - m = p - 1 \quad (103)$$

$$n = m + p - 1 \quad (104)$$

Where T_n is the value of the n^th^ term, f is the first term, k is the migration level value, i is the migration step value, n is the number of terms, a is the migration level factor, m is migration step factor respectively, l is the level value of the term and p is the position of the n^th^ term in the Kifilideen's Matrix Structural Framework.

From (76), $= m = \frac{a^2+a+2}{2} = \frac{c^2-c+2}{2}$ (105)

Put (88) in (104), we have:

$$n = \frac{a^2+a+2}{2} + p - 1 \quad (106)$$

$$n = \frac{a^2+a+2+2p-2}{2} \quad (107)$$

$$n = \frac{a^2+a+2p}{2} \quad (108)$$

OR

Equation (108) can be inaugurated as follows:

Table 6 shows the relationship between $l, p, n,$ and a of the infinite term of Kifilideen's Generalized Matrix Progression Sequence of increasing members of successive levels and one member in the first level.

From Table 6; we have:

Position 1

l=1, a=0; n=1=1 (109)
l=2, a=1; n=1+1=2 (110)
l=3, a=2; n=1+1+2=4 (111)
l=4, a=3; n=1+1+2+3=7 (112)
l=5, a=4; n=1+1+2+3+4=11 (113)
l=6, a=5; n=1+1+2+3+4+5=16 (114)

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Position 2

l=2, a=1; n=2+1=3 (115)
l=3, a=2; n=2+1+2=5 (116)
l=4, a=3; n=2+1+2+3=8 (117)
l=5, a=4; n=2+1+2+3+4=12 (118)
l=6, a=5; n=2+1+2+3+4+5=17 (119)

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Position 3

l=3, a=2; n=3+1+2=6 (120)
l=4, a=3; n=3+1+2+3=9 (121)
l=5, a=4; n=3+1+2+3+4=13 (122)
l=6, a=5; n=3+1+2+3+4+5=18 (123)

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Position 4

l=4, a=3; n=4+1+2+3=10 (124)
l=5, a=4; n=4+1+2+3+4=14 (125)
l=6, a=5; n=4+1+2+3+4+5=19 (126)

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Position p

l=l,a=a; n=p+1+2+3+4+5+⋯+a (127)

Let n=p+τ in the series of n in (127)
Where τ=1+2+3+4+5+⋯ (128)

The series of τ is an arithmetic progression series. The summation of the arithmetic progression series presented by Bunday and Mulhollsnd (2014); and Godman et al. (1984) is utilized as follows:

$$S_v = \frac{v}{2}(2y + (v-1)d) \quad (129)$$

Where S_v is sum of the series, τ, v is the number of terms of the series, τ, y is the first term, and d is the common difference between two successive terms of the series, τ.

From (128),

$S_v = \tau, v = a$ =Migration level factor, $y = 1$,

$$d = T_2 - T_1 = 2 - 1 = 1 \quad (130)$$

$$S_v = \tau = \frac{a}{2}(2 \times 1 + (a-1) \times 1) \quad (131)$$

$$\tau = \frac{a}{2}(2 + a - 1) \quad (132)$$

Table 6. The relationship between l, p, n and a of the infinite term.

l	a	p	n
1	0	**1**	**1** = 1
2	1	**1**	**1** + 1 = 2
2	1	**2**	**2** + 1 = 3
3	2	**1**	**1** + 1 + 2 = 4
3	2	**2**	**2** + 1 + 2 = 5
3	2	**3**	**3** + 1 + 2 = 6
4	3	**1**	**1** + 1 + 2 + 3 = 7
4	3	**2**	**2** + 1 + 2 + 3 = 8
4	3	**3**	**3** + 1 + 2 + 3 =9
4	3	**4**	**4** + 1 + 2 + 3 =10
.	.	.	.
.	.	.	.
.	.	.	.
l	$a = l - 1$	$\boldsymbol{p}$	$\boldsymbol{p} + 1 + 2 + 3 + 4 + 5 + \cdots + \cdots + \cdots + a$
.	.	.	.
.	.	.	.
.	.	.	.

$$\tau = \frac{a(a+1)}{2} \quad (133)$$

Put (133) in (128), we have:

$$n = \boldsymbol{p} + \tau = \boldsymbol{p} + \frac{a(a+1)}{2} \quad (134)$$

$$n = \frac{2p + a^2 + a}{2} \quad (135)$$

$$n = \frac{a^2 + a + 2p}{2} \quad (136)$$

Where n the number of terms, a is the migration level factor and p is the position of the nth term in the Kifilideen's Matrix Structural Framework.

This (136) is the same as what was obtained in (108).

2.6. Kifilideen's Term Step Level Mathematical Formula of the Infinite Term

Table 7 layouts the step and level of each term of the infinite term of Kifilideen's Generalized Matrix Progression Sequence of increasing members of successive levels with one member in the first level in Kifilideen's matrix structural framework. In level 1: T_1 is in step level (s, l): (1,1); in level 2: T_2, T_3 are in step level (s, l): (2,2) and (3,2) respectively; in level 3: T_4, T_5, $T_6$are in step level (s, l): (3,3), (4,3) and (5,3) respectively; in level 4: T_7, T_8, T_9, T_{10} are in step level (s, l): (4,4), (5,4), (6,4) and (7,4) respectively; in level 5: T_{11}, T_{12}, T_{13}, T_{14}, T_{15} are in step level (s, l): (5,5), (6,5), (7,5), (8,5) and (9,5) respectively; …, …, …,

The stepwise analysis of the mathematical induction of the Kifilideen's term step level mathematical formula of the infinite term of the Kifilideen's Generalized Matrix Progression Sequence of the infinite term of increasing members of successive levels with one member in the first level is presented as:

Step 1, **Level** 1; sl_{11}, $T_1 = k(0) + i(0) + f$ (137)
Step **1**, Level **1**; sl_{11}, $T_1 = k(\mathbf{1} - 1) + i(\mathbf{1} - \mathbf{1}) + f$ (138)
Step 2, **Level** 2; sl_{22}, $T_2 = k(1) + i(0) + f$ (139)
Step **2**, Level **2**; sl_{22}, $T_2 = k(\mathbf{2} - 1) + i(\mathbf{2} - \mathbf{2}) + f$ (140)
Step 3, Level 2; sl_{32}, $T_3 = k(1) + i(1) + f$ (141)
Step **3**, Level **2**; sl_{32}, $T_3 = k(\mathbf{2} - 1) + i(\mathbf{3} - \mathbf{2}) + f$ (142)
Step 3, **Level** 3; sl_{33}, $T_4 = k(2) + i(0) + f$ (143)
Step **3**, Level **3**; sl_{33}, $T_4 = k(\mathbf{3} - 1) + i(\mathbf{3} - \mathbf{3}) + f$ (144)
Step 4, Level 3; sl_{43}, $T_5 = k(2) + i(0) + f$ (145)
Step **4**, Level **3**; sl_{43}, $T_5 = k(\mathbf{3} - 1) + i(\mathbf{4} - \mathbf{3}) + f$ (146)
Step 5, Level 3; sl_{53}, $T_6 = k(2) + i(0) + f$ (147)
Step **5**, Level **3**; sl_{53}, $T_6 = k(\mathbf{3} - 1) + i(\mathbf{5} - \mathbf{3}) + f$ (148)
Step 4, **Level** 4; sl_{44}, $T_7 = k(3) + i(0) + f$ (149)
Step **4**, Level **4**; sl_{44}, $T_7 = k(\mathbf{4} - 1) + i(\mathbf{4} - \mathbf{4}) + f$ (150)
Step 5, Level 4; sl_{54}, $T_8 = k(3) + i(1) + f$ (151)
Step **5**, Level **4**; sl_{54}, $T_8 = k(\mathbf{4} - 1) + i(\mathbf{5} - \mathbf{4}) + f$ (152)
Step 6, Level 4; sl_{64}, $T_9 = k(3) + i(2) + f$ (153)
Step **6**, Level **4**; sl_{64}, $T_9 = k(\mathbf{4} - 1) + i(\mathbf{6} - \mathbf{4}) + f$ (154)
Step 7, Level 4; sl_{74}, $T_{10} = k(3) + i(3) + f$ (155)
Step **7**, Level **4**; sl_{74}, $T_{10} = k(\mathbf{4} - 1) + i(\mathbf{7} - \mathbf{4}) + f$ (156)

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Step ***s***, Level ***l***; sl_{rc}, $T_n = k(\boldsymbol{l} - 1) + i(\boldsymbol{s} - \boldsymbol{l}) + f$ (157)

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Table 7: The layout of the step and level of each term of the infinite term

	l_1	l_2	l_3	l_4	l_5	...
s_1	$sl_{11} \to T_1$					
s_2		$sl_{22} \to T_2$				
s_3		$sl_{32} \to T_3$	$sl_{33} \to T_4$			
s_4			$sl_{43} \to T_5$	$sl_{44} \to T_7$		
s_5			$sl_{53} \to T_6$	$sl_{54} \to T_8$	$sl_{55} \to T_{11}$	
s_6				$sl_{64} \to T_9$	$sl_{65} \to T_{12}$	
s_7				$sl_{74} \to T_{10}$	$sl_{75} \to T_{13}$	.
s_8					$sl_{85} \to T_{14}$	..
s_9					$sl_{95} \to T_{15}$	...
s_{10}						...
s_{11}						...
s_{12}						...

Generally, the Kifilideen's term step level mathematical formula of the Kifilideen's Generalized Matrix Progression Sequence of the infinite term of increasing members of successive levels with one member in the first level is expressed as:

$$T_n = k(\boldsymbol{l} - 1) + i(\boldsymbol{s} - \boldsymbol{l}) + f \quad (158)$$

Comparing (74) with (158); we have:

$$n - m = s - l \quad (159)$$

$$n = m + s - l \quad (160)$$

From (76), $m = \frac{l^2 - l + 2}{2}$ (161)

Put (76) in (160); we have:

$$n = \frac{l^2 - l + 2}{2} + s - l \quad (162)$$

$$n = \frac{l^2 - l + 2 + 2s - 2l}{2} \quad (163)$$

$$n = \frac{l^2 - 3l + 2s + 2}{2} \quad (164)$$

$2n = l^2 - 3l + 2s + 2$ (165)

$s = \frac{2n - l^2 + 3s - 2}{2}$ (166)

Note: In any particular level of the Kifilideen's Generalized Matrix Progression Sequence of the infinite term of increasing members of successive levels with one member in the first level in the Kifilideen's Matrix Structural Framework,

$$s \geq l$$

Also, in any particular level of the Kifilideen's Generalized Matrix Progression Sequence of the infinite term of increasing members of successive levels with one member in the first level in the Kifilideen's Matrix Structural Framework, the level accommodates members until the coefficient of k and i are the same.

From (158), $T_n = k(l-1) + i(s-l) + f$ (167)

So, at the last member of a particular level in the Kifilideen's Matrix Structural Framework of the Kifilideen's Generalized Matrix Progression Sequence of the infinite term of increasing members of successive levels with one member in the first level the coefficient of k and i are the same.

Therefore we have: $l - 1 = s - l$ (168)

$s = 2l - 1$ (169)

So, the maximum value of step, s is obtained for any particular level, l using the (169).

At the start of any step in the Kifilideen's Matrix Structural Framework of the Kifilideen's Generalized Matrix Progression Sequence of the infinite term of increasing members of successive levels with one member in the first level, the first member in that level is in equal value of step and level that is:

$s = l$ (170)

Comparing (102) with (167); we have:

$p - 1 = s - l$ (171)

$p = s - l + 1$ (172)

2.7. Kifilideen's Step–Level Mathematical Formula of the Infinite Term

Table 8 presents the layout of the step–level of each term of the infinite term of the Kifilideen's Generalized Matrix Progression Sequence of the infinite term of increasing members of successive levels with one member in the first level in the Kifilideen's Matrix Structural Framework. In level 1: T_1 is in step level (sl): 11; in level 2: T_2, T_3 are in step level (sl): 22 and 32 respectively; in level 3: T_4, T_5, T_6 are in step level (sl): 33, 43 and 53 respectively; in level 4: T_7, T_8, T_9, T_{10} are in step level (sl): 44, 54, 64 and 74 respectively; in level 5: T_{11}, T_{12}, T_{13}, T_{14}, T_{15} are in step level (sl): 55, 65, 75, 85 and 95 respectively; ..., ..., ...,

Table 8. The layout of the step and level of each term of the infinite term

	l_1	l_2	l_3	l_4	l_5	...
s_1	$11 \rightarrow T_1$					
s_2		$22 \rightarrow T_2$				
s_3		$32 \rightarrow T_3$	$33 \rightarrow T_4$			
s_4			$43 \rightarrow T_5$	$44 \rightarrow T_7$		
s_5			$53 \rightarrow T_6$	$54 \rightarrow T_8$	$55 \rightarrow T_{11}$	
s_6				$64 \rightarrow T_9$	$65 \rightarrow T_{12}$	
s_7				$74 \rightarrow T_{10}$	$75 \rightarrow T_{13}$	.
s_8					$85 \rightarrow T_{14}$	..
s_9					$95 \rightarrow T_{15}$	...
s_{10}						...
s_{11}						...
s_{12}						...

The stepwise analysis of the mathematical induction of Kifilideen's step–level mathematical formula of the infinite term of Kifilideen's Generalized Matrix Progression Sequence of the infinite term of increasing members of successive levels with one member in the first level is generated as follows: The arrangement of the step and level sl in Kifilideen's Matrix Structural Framework is in the form of an infinite term of Kifilideen's Generalized Matrix Progression Sequence of increasing members of successive levels with one member in the first level. In Table 8,

f =first term $= 11$, (173)

k =migration level value $= T_2 - T_1 = 22 - 11 = 11$ (174)

i =migration step value $= T_3 - T_2 = 32 - 22 = 10$ (175)

Where f is the first term, k is the migration level value, i is the migration step value, T_1 is the value of the first term, T_2 is the value of the second term and T_3 is the value of the third term of Kifilideen's Matrix Structural Framework of Table 8.

The stepwise analysis of the mathematical induction of Kifilideen's **step–level** mathematical formula of Kifilideen's Generalized Matrix Progression Sequence of the infinite term of increasing members of successive levels with one member in the first level is established. Using Kifilideen's term formula invented in (50) for the infinite term of Kifilideen's Generalized Matrix Progression Sequence of the infinite term of increasing members of successive levels with one member in the first level, we have:

Step 1 Level 1;	T_1;	$sl = 11(0) + 10(0) + 11 = 11$	(176)
Step 1 Level 1;	$T_{\mathbf{1}}$;	$sl = 11(0) + 10(\mathbf{1} - 1) + 11 = 11$	(177)
Step 2 Level 2;	T_2;	$sl = 11(1) + 10(0) + 11 = 22$	(178)
Step 2 Level 2;	$T_{\mathbf{2}}$;	$sl = 11(1) + 10(\mathbf{2} - 2) + 11 = 22$	(179)
Step 3 Level 2;	T_3;	$sl = 11(1) + 10(1) + 11 = 23$	(180)
Step 3 Level 2;	$T_{\mathbf{3}}$;	$sl = 11(1) + 10(\mathbf{3} - 2) + 11 = 23$	(181)
Step 3 Level 3;	T_4;	$sl = 11(2) + 10(0) + 11 = 33$	(182)
Step 3 Level 3;	$T_{\mathbf{4}}$;	$sl = 11(2) + 10(\mathbf{4} - 4) + 11 = 33$	(183)
Step 4 Level 3;	T_5;	$sl = 11(2) + 10(1) + 11 = 43$	(184)
Step 4 Level 3;	$T_{\mathbf{5}}$;	$sl = 11(2) + 10(\mathbf{5} - 4) + 11 = 43$	(185)
Step 5 Level 3;	T_6;	$sl = 11(2) + 10(2) + 11 = 53$	(186)
Step 5 Level 3;	$T_{\mathbf{6}}$;	$sl = 11(2) + 10(\mathbf{6} - 4) + 11 = 53$	(187)
Step 4 Level 4;	T_7;	$sl = 11(3) + 10(0) + 11 = 44$	(188)
Step 4 Level 4;	$T_{\mathbf{7}}$;	$sl = 11(3) + 10(\mathbf{7} - 7) + 11 = 44$	(189)
Step 5 Level 4;	T_8;	$sl = 11(3) + 10(1) + 11 = 54$	(190)
Step 5 Level 4;	$T_{\mathbf{8}}$;	$sl = 11(3) + 10(\mathbf{8} - 7) + 11 = 54$	(191)
Step 6 Level 4;	T_9;	$sl = 11(3) + 10(2) + 11 = 64$	(192)
Step 6 Level 4;	$T_{\mathbf{9}}$;	$sl = 11(3) + 10(\mathbf{9} - 7) + 11 = 64$	(193)
Step 7 Level 4;	T_{10};	$sl = 11(3) + 10(3) + 11 = 74$	(194)
Step 7 Level 4;	$T_{\mathbf{10}}$;	$sl = 11(3) + 10(\mathbf{10} - 7) + 11 = 74$	(195)

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Step s Level l;	$T_{\boldsymbol{n}}$;	$sl = 11(a) + 10(\boldsymbol{n} - m) + 11 = sl$	(196)

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Generally, Kifilideen's step–level mathematical formula for the infinite term of the Kifilideen's Generalized Matrix Progression Sequence of the infinite term of increasing members of successive levels with one member in the first level is invented as:

$$sl = 11(a) + 10(n - m) + 11 \quad (197)$$

Where n is the number of terms, a is the migration level factor, m is the migration step factor, and sl is the step–a level value of the term.

3. Results and Discussion

If an agricultural processing company wishes to adopt Kifilideen's Generalized Matrix Progression Sequence of infinite terms to develop a salary structure for the staff of the company, the staff of the company in the 5^{th}, 10^{th} and 13^{th} terms of Kifilideen's Structural Matrix Framework receive monetary incentives of ₦95,000, ₦125,000 and ₦140,000 respectively. Determine the following:

(i) The level, the step, and the position of the staff in the $5^{th}, 10^{th}$ and 13^{th} terms of the Kifilideen's Structural Matrix Framework;
(ii) The migration level value of the salary structure;
(iii) The migration step value of the salary structure;
(iv) The salary received by the staff in the first term of the Kifilideen's Structural Matrix Framework;
(v) Find the salary to be received by staff in the 15^{th} term of the Kifilideen's Structural Matrix Framework; also, state the level, the step, and the position of the term of the staff;
(vi) Produce Kifilideen's Structural Matrix Framework for the salary structure of the company for the first six levels.

Solution

1(a) the level, the step, and the position of the staff in the 5^{th} term of the Kifilideen's Structural Matrix Framework is obtained as follows:

For 5^{th} term, $n = 5$, $T_5 = ₦\,95,000$ (198)

The migration level factor, a of the staff in the 5^{th} the term is achieved as:

Migration level factor of the staff in the 5^{th} term $=$

$$a = \frac{-1+\sqrt{8n-7}}{2} \quad (199)$$

Migration level factor of the staff in the 5^{th} term $=$

$$a = \frac{-1+\sqrt{8\times5-7}}{2} \quad (200)$$

Migration level factor of the staff in the 5^{th} term $=$

$$a = 2.3723 \quad (201)$$

Migration level factor of the staff in the 5^{th} term $=$

$$a = 2 \quad (202)$$

So, the migration level factor, a of the staff in 5^{th} term = 2

For the migration step factor, m of the staff in the 5^{th} term, we have:

Migration step factor of the staff in the 5^{th} term $=$

$$m = \frac{a^2+a+2}{2} \quad (203)$$

Migration step factor of the staff in the 5^{th} term $=$

$$m = \frac{2^2+2+2}{2} \quad (204)$$

Migration step factor of the staff in the 5^{th} term $=$

$$m = 4 \quad (205)$$

So, the migration step factor, m of the staff in 5^{th} term is 4.

The migration level factor, a of the staff in the 5^{th} term obtained in (202) is used to obtain the value of the level of the staff in the 5^{th} term which is presented as follows:

The level of the staff in the 5^{th} term =

$$l = a + 1 = 2 + 1 = 3 \quad (206)$$

So, the staff in $5^{th} the$ term is in level 3.

The value of the level of the staff in the 5^{th} term attained in (206) and the number of terms of the staff in the 5^{th} term is used to determine the value of the step of the staff in the 5^{th} a term which is obtained as follows:

Step of the staff in the 5^{th} term =

$$s = \frac{2n-l^2+3l-2}{2} \quad (207)$$

Step of the staff in the 5^{th} term =

$$s = \frac{2\times5-3^2+3\times3-2}{2} \quad (208)$$

Step of the staff in the 5^{th} term $= 4$ (209)

Therefore, the staff in the 5^{th}term is in step 4.

The number of terms and the migration step factor, m of the staff in the 5^{th} term is used to determine the value of the position of the staff in the 5^{th} term which is presented as follows:

Position of the staff in the 5^{th} term=

$$n - m + 1 \quad (210)$$

Position of the staff in the 5^{th} term =

$$5 - 4 + 1 \quad (211)$$

Position of the staff in the 5^{th} term $= 2$ (212)

So, the staff in the $5^{th} the$ term is in position 2.

The equation generated for the 5^{th}term using the Kifilideen's term formula of the Kifilideen's Generalized matrix progression sequence is presented as follows:

$$T_n = k(a) + i(n - m) + f \quad (213)$$

Number of terms, n of the staff in the $5^{th} the$ term is 5, the migration level factor, a of the staff in the 5^{th} term is 2 and the migration step factor, m of the staff in the 5^{th} term is 4. From the question [1] the staff in the 5^{th} the term received monetary incentives of ₦ $95,000$, so the equation generated for the $5^{th} the$ term is given as:

Fifth term $T_5 = k(2) + i(5 - 4) + f = ₦\,95,000$ (214)

$2k + i + f = ₦\,95,000$ (215)

From (206), (209) and (212), the staff in the 5^{th} the term is in level 3, step 4, and position 2 in the Kifilideen's Structural Matrix Framework

1(ib) The level, the step, and the position of the staff in the 10^{th} term of the Kifilideen's Structural Matrix Framework is obtained as follows:

For 10^{th} term, $n = 10$, $T_{10} = ₦\,125,000$ (216)

The migration level factor, a of the staff in the 10^{th} the term is achieved as:

Migration level factor of the staff in the 10^{th} term =

$$a = \frac{-1+\sqrt{8n-7}}{2} \quad (217)$$

Migration level factor of the staff in the 10^{th} term =

$$a = \frac{-1+\sqrt{8\times10-7}}{2} \quad (218)$$

Migration level factor of the staff in the 10^{th} term =

$a = 3.7720$ (219)

Migration level factor of the staff in the 10^{th} term =

$a = 3$ (220)

So, the migration level factor, a of the staff in the 10^{th} term is 3.

For the migration step factor, m of the staff in the 10^{th} term, we have:

Migration step factor of the staff in the 10^{th} term =

$$m = \frac{a^2+a+2}{2} \quad (221)$$

Migration step factor of the staff in the 10^{th} term =

$$m = \frac{3^2+3+2}{2} \quad (222)$$

Migration step factor of the staff in the 10^{th} term =

$m = 7$ (223)

So, the migration step factor, m of the staff in the 10^{th} term is 7.

The migration level factor, a of the staff in the 10^{th} term obtained in (220) is used to obtain the value of the level of the staff in the 10^{th} term which is presented as follows:

The level of the staff in the 10^{th} term =

$l = a + 1 = 3 + 1 = 4$ (224)

So, the staff in $10^{th}$$the$ term is in level 4.

The value of the level of the staff in the 10^{th} term attained in (224) and the number of terms of the staff in the 10^{th} term is used to determine the value of the step of the staff in the 10^{th} the term which is obtained as follows:

Step of the staff in the 10^{th} term =

$$s = \frac{2n-l^2+3l-2}{2} \quad (225)$$

Step of the staff in the 10^{th} term =

$$s = \frac{2\times10-4^2+3\times4-2}{2} \quad (226)$$

Step of the staff in the 10^{th} term = 7 (227)

Therefore, the staff in the 10^{th}term is in step 7.

The number of terms and the migration step factor, m of the staff in the 10^{th} term is used to determine the value of the position of the staff in the 10^{th} term which is presented as follows:

Position of the staff in the 10^{th} term= $n - m + 1$ (228)

Position of the staff in the 10^{th} term = $10 - 7 + 1$(229)

Position of the staff in the 10^{th} term = 4 (230)

So, the staff in the $10^{th}$$the$ term is in position 4.

The equation generated for the 10^{th}term using the Kifilideen's term formula of the Kifilideen's Generalized matrix progression sequence is presented as follows:

$$T_n = k(a) + i(n-m) + f \quad (231)$$

Number of terms, n of the staff in the $10^{th}$$the$ term is 10, the migration level factor, a of the staff in the 10^{th} term is 3 and the migration step factor, m of the staff in the 10^{th} term is 7. From the question [1] the staff in the 10^{th} term received monetary incentives of $₦\,125,000$, so the equation generated for the $10^{th}$$the$ term is given as:

Tenth term $T_{10} = k(3) + i(10-7) + f = ₦125,000$ (232)

$3k + 3i + f = ₦\,125,000$ (233)

From (224), (227) and (230), the staff in the 10^{th} the term is in level 4, step 7 and position 4 in Kifilideen's Structural Matrix Framework.

1(ic) The level, the step and the position of the staff in the 13^{th} term of the Kifilideen's Structural Matrix Framework is obtained as follows:

For 13^{th} term, $n = 13$, $T_{13} = ₦\,140,000$ (234)

The migration level factor, a of the staff in the 13^{th} term is achieved as:

Migration level factor of the staff in the 13^{th} term =

$$a = \frac{-1+\sqrt{8n-7}}{2} \quad (235)$$

Migration level factor of the staff in the 13^{th} term =

$$a = \frac{-1+\sqrt{8\times13-7}}{2} \quad (236)$$

Migration level factor of the staff in the 13^{th} term =

$$a = 4.4244 \quad (237)$$

Migration level factor of the staff in the 13^{th} term =

$$a = 4 \quad (238)$$

So, the migration level factor, a of the staff in the 13^{th} term is 4.

For the migration step factor, m of the staff in the 13^{th} term, we have:

Migration step factor of the staff in the 13^{th} term

$$m = \frac{a^2+a+2}{2} \quad (239)$$

Migration step factor of the staff in the 13^{th} term =

$$m = \frac{4^2+4+2}{2} \quad (240)$$

Migration step factor of the staff in the 13^{th} term =

$$m = 11 \quad (241)$$

So, the migration step factor, m of the staff in the 13^{th} term is 11.

The migration level factor, a of the staff in the 13^{th} term obtained in (238) is used to obtain the value of the level of the staff in the 13^{th} term which is presented as follows:

Level of the staff in the 13^{th} term =

$$l = a + 1 = 4 + 1 = 5 \quad (242)$$

So, the staff in 13^{th} the term is in level 5.

The value of the level of the staff in the 13^{th} term attained in (242) and the number of terms of the staff in the 13^{th} term is used to determine the value of the step of the staff in the 13^{th} a term which is obtained as follows:

Step of the staff in the 13^{th} term =

$$s = \frac{2n-l^2+3l-2}{2} \quad (243)$$

Step of the staff in the 13^{th} term =

$$s = \frac{2\times13-5^2+3\times5-2}{2} \quad (244)$$

Step of the staff in the 13^{th} term $= 7$ (245)

Therefore, the staff in the 13^{th}term is in step 7.

The number of terms and the migration step factor, m of the staff in the 13^{th} term is used to determine the value of the position of the staff in the 13^{th} term which is presented as follows:

Position of the staff in the 13^{th} term =

$$p = n - m + 1 \quad (246)$$

Position of the staff in the 13^{th} term =

$$p = 13 - 11 + 1 \quad (247)$$

Position of the staff in the 13^{th} term $= p = 3$ (248)

So, the staff in the 10^{th} the term is in position 3.

The equation generated for the 13^{th}term using the Kifilideen's term formula of the Kifilideen's generalized matrix progression sequence is presented as follows:

$$T_n = k(a) + i(n-m) + f \quad (249)$$

Number of terms, n of the staff in the 13^{th} the term is 13, the migration level factor, a of the staff in the 13^{th} term is 4 and the migration step factor, m of the staff in the 13^{th} term is 11. From the question [1] the staff in the 13^{th} the term received monetary incentives of ₦140,000, so the equation generated for the 13^{th} the term is given as:

Thirteenth term =

$$T_{13} = k(4) + i(13-11) + f = ₦140,000 \quad (250)$$

$$4k + 2i + f = ₦140,000 \quad (251)$$

From (242), (245) and (248), the staff in the 13^{th} term is in level 5, step 7 and position 3 in Kifilideen's Structural Matrix Framework

1(ii) the value of the migration level value, k, the migration step value, i, and the salary received by the staff in the first term, f of the salary structure is obtained as follows:

From (215), (233), and (251), we have:

$$2k + i + f = ₦95,000 \quad (252)$$

$$3k + 3i + f = ₦125,000 \quad (253)$$

$$4k + 2i + f = ₦140,000 \quad (254$$

Using Crammer's rule, we have:

$$\begin{pmatrix} 2 & 1 & 1 \\ 3 & 3 & 1 \\ 4 & 2 & 1 \end{pmatrix} \begin{pmatrix} k \\ i \\ f \end{pmatrix} = \begin{pmatrix} ₦95,000 \\ ₦125,000 \\ ₦140,000 \end{pmatrix} \quad (255)$$

$$\Delta = \begin{vmatrix} 2 & 1 & 1 \\ 3 & 3 & 1 \\ 4 & 2 & 1 \end{vmatrix} = 2\begin{vmatrix} 3 & 1 \\ 2 & 1 \end{vmatrix} - 1\begin{vmatrix} 3 & 1 \\ 4 & 1 \end{vmatrix} + 1\begin{vmatrix} 3 & 3 \\ 4 & 2 \end{vmatrix} = 2(3-2) - 1(3-4) + 1(6-12) \quad (256)$$

$$\Delta = 2(1) - 1(-1) + (-6) = 2 + 1 - 6 = -3 \quad (257)$$

$$\Delta k = \begin{vmatrix} 95,000 & 1 & 1 \\ 125,000 & 3 & 1 \\ 140,000 & 2 & 1 \end{vmatrix} = 95,000\begin{vmatrix} 3 & 1 \\ 2 & 1 \end{vmatrix} - 1\begin{vmatrix} 125,000 & 1 \\ 140,000 & 1 \end{vmatrix} + 1\begin{vmatrix} 125,000 & 3 \\ 140,000 & 2 \end{vmatrix} \quad (258)$$

$$\Delta k = 95,000(3-2) - 1(125000 - 140000) + (250000 - 420000) \quad (259)$$

$$\Delta k = 95,000(1) - 1(-15,000) + (-170,000) = -₦60,000 \quad (260)$$

$$\Delta i = \begin{vmatrix} 2 & 95,000 & 1 \\ 3 & 125,000 & 1 \\ 4 & 140,000 & 1 \end{vmatrix} = 2\begin{vmatrix} 125,000 & 1 \\ 140,000 & 1 \end{vmatrix} - 95,000\begin{vmatrix} 3 & 1 \\ 4 & 1 \end{vmatrix} + 1\begin{vmatrix} 3 & 125,000 \\ 4 & 140,000 \end{vmatrix} \quad (261)$$

$$\Delta i = 2(125,000 - 140,000) - 95,000(3-4) + 1(420,000 - 500,000) \quad (262)$$

$$\Delta i = 2(-15,000) - 95,000(-1) + 1(-80,000) = -30,000 + 95,000 - 80,000 = -15,000 \quad (263)$$

$$\Delta f = \begin{vmatrix} 2 & 1 & 95,000 \\ 3 & 3 & 125,000 \\ 4 & 2 & 140,000 \end{vmatrix} = 2\begin{vmatrix} 3 & 125,000 \\ 2 & 140,000 \end{vmatrix} - 1\begin{vmatrix} 3 & 125,000 \\ 4 & 140,000 \end{vmatrix} + 95,000\begin{vmatrix} 3 & 3 \\ 4 & 2 \end{vmatrix} \quad (264)$$

$$\Delta f = 2(420,000 - 250,000) - 1(420,000 - 500,000) + 95,000(6-12) \quad (265)$$

$$\Delta f = 2(170,000) - 1(-80,000) + 95,000(-6) = 340,000 + 80,000 - 570,000 = -₦150,000 \quad (266)$$

$k = \frac{\Delta k}{k} = \frac{-₦60,000}{-3} = ₦20,000;$ (267)

$i = \frac{\Delta k}{k} = \frac{-₦15,000}{-3} = ₦5,000;$ (268)

$f = \frac{\Delta k}{k} = \frac{-₦150,000}{-3} = ₦50,000.$ (269)

1(ii) the migration level value of the salary structure = $k = ₦20,000;$ (270)

1(iii) the migration step value of the salary structure = $i = ₦5,000;$ (271)

1(iv) the salary received by the staff in the first term = $f = ₦50,000.$ (272)

1(v) For 15^{th} the term, $n = 15,$ (273)

The migration level factor, a of the staff in the 15^{th} the term is achieved as:

Migration level factor of the staff in the 15^{th} term = $a = \frac{-1+\sqrt{8n-7}}{2}$ (274)

Migration level factor of the staff in the 15^{th} term = $a = \frac{-1+\sqrt{8\times 15-7}}{2}$ (275)

Migration level factor of the staff in the 15^{th} term = $a = \mathbf{4}.8151$ (276)

Migration level factor of the staff in the 15^{th} term = $a = 4$ (277)

So, the migration level factor, a of the staff in the 15^{th} term is 4.

For the migration step factor, m of the staff in the 15^{th} term, we have:

Migration step factor= $m = \frac{a^2+a+2}{2}$ (278)

Migration step factor= $m = \frac{4^2+4+2}{2}$ (279)

Migration step factor= $m = 11$ (280)

So, the migration step factor, m of the staff in the 15^{th} term is 11.

The salary to be received by staff in the 15^{th} the term is obtained as:

$T_n = k(a) + i(n-m) + f$ (282)

Fifteenth term = $T_{15} = 20,000 \times (4) + 5,000 \times (15-11) + 50,000$ (283)

T_{15} = the salary received by staff in the 15^{th} term $= ₦150,000$ (284)

Level of the staff in the 15^{th} term = $l = a + 1 = 4 + 1 = 5$ (285)

So, the staff in $15^{th} the$ term is in level 5.

Step of the staff in the 15^{th} term =

$$s = \frac{2n - l^2 + 3l - 2}{2} \quad (286)$$

Step of the staff in the 15^{th} term =

$$s = \frac{2 \times 15 - 5^2 + 3 \times 5 - 2}{2} \quad (287)$$

Step of the staff in the 15^{th} term $= 9$ (288)

Therefore, the staff in the $15^{th} the$ term is in step 9.

Position of the staff in the 15^{th} term=

$$p = n - m + 1 \quad (289)$$

Position of the staff in the 15^{th} term =

$$p = 15 - 11 + 1 \quad (290)$$

Position of the staff in the 15^{th} term =

$$p = 5 \quad (291)$$

So, the staff in the $15^{th} the$ term is in position 5.

From (285), (288) and (291), the staff in the 15^{th} the term is in level 5, step 9 and position 5 in Kifilideen's Structural Matrix Framework

1(vi) To produce Kifilideen's Structural Matrix Framework for the salary structure of the company for the first five levels we have:

The migration level value of the salary structure = $k =$ ₦ $20,000$; the migration step value of the salary structure $= i =$ ₦ $5,000$, and the first term $= f =$ ₦ $50,000$. So, Kifilideen's Structural Matrix Framework for the salary structure of the company for the first five levels is presented in Table 9.

Table 9. The salary structure of the company of question 1 for the first five levels.

	l_1	l_2	l_3	l_4	l_5
s_1	$T_1 =$ ₦ $50,000$				
s_2		$T_2 =$ ₦ $70,000$			
s_3		$T_3 =$ ₦ $75,000$	$T_4 =$ ₦ $90,000$		
s_4			$T_5 =$ ₦ $95,000$	$T_7 =$ ₦ $110,000$	
s_5			$T_6 =$ ₦ $100,000$	$T_8 =$ ₦ $115,000$	$T_{11} =$ ₦ $130,000$
s_6				$T_9 =$ ₦ $120,000$	$T_{12} =$ ₦ $135,000$
s_7				$T_{10} =$ ₦ $125,000$	$T_{13} =$ ₦ $140,000$
s_8					$T_{14} =$ ₦ $145,000$
s_9					$T_{15} =$ ₦ $150,000$

From Table 9, the salary structure increases from one level to another towards the right and also increases down within a level. Table 9 indicates that the migration level value is ₦ $20,000$ and the migration step value is ₦ $5,000$. Also, from the Table 9, it is noted that the difference between the salaries received by the last member in a level and the last member in the successive level is ₦ $25,000$. This is obtained by the addition of the migration level value, k and the migration step value, i.

[2] A developer develops a balloon blow–up video game adopting Kifilideen's Generalized Matrix Progression Sequence of infinite terms. To migrate from one level and step to another, the player has to blow up a definite number of balloons following the progression sequence. If the number of balloons in the 1^{st}, 2^{nd} and 3^{rd} terms of the game are 10, 27 and 35 respectively, determine the following:

(i) The migration level value of the game;

(ii) The migration level value of the game;

(iii) The first term value of the game;

(iv) The number of balloons in level 4 and step 6 of the game and the number of terms at this stage;

(v) The number of balloons in 7^{th} term of the game;

(vi) If the number of balloons in level 5 is 86, find the step the player would be in at such level. Also, determine the number of terms the player would be in in the game.

(vii) Produce Kifilideen's Structural Matrix Framework for the balloon blow–up video game for the first five levels.

Solution

From question [2], we have: $T_1 = 10, T_2 = 27, T_3 = 35$

2(i) the migration level value of the game =

$$k = T_2 - T_1 = 27 - 10 = 17 \quad (292)$$

2(ii) the migration step value of the game =

$i = T_3 - T_2 = 35 - 27 = 8$ (293)

2(iii) the first term value of the game $= f = 10,$ (294)

2(iv) To find the number of balloons in level 4 and step 6 of the game, we have:

$T_n = k(l-1) + i(s-l) + f$ (295)

$l = 4, s = 6, k = 17, i = 8$ and $f = 10$ (296)

$T_n = 17 \times (4-1) + 8 \times (5-4) + 11 = 7 \times 3 + 8 \times 1 + 11 = 40$ (297)

The number of balloons in level 4 and step 6 of the game is 40.

The number of terms of the game at this stage $= n = \frac{l^2-3l+2s+2}{2} = \frac{4^2-3\times4+2\times6+2}{2} = 9$ (298)

The stage of the game is 9 th term.

2(v) to find the number of balloons in 7 th term of the game, we have:

For 7 th the term, $n = 7,$ (299)

Migration level factor of the number of balloons in the 7 th term $= a = \frac{-1+\sqrt{8n-7}}{2}$ (300)

Migration level factor of the number of balloons in the 7 th term$= a = \frac{-1+\sqrt{8\times7-7}}{2}$ (301)

Migration level factor of the number of balloons in the 7 th term $= a = \mathbf{3}$ (302)

Migration level factor of the number of balloons in the 7 th term$= a = 3$ (303)

Migration step factor of the number of balloons in the 7 th term $= m = \frac{a^2+a+2}{2}$ (304)

Migration step factor of the number of balloons in the 7 th term $= m = \frac{3^2+3+2}{2}$ (305)

Migration step factor of the number of balloons in the 7 th term $= m = 7$ (306)

The number of balloons in the 7 th term of the game is obtained as:

$T_n = k(a) + i(n-m) + f$ (307)

Seventh term $= T_7 = 17 \times (3) + 8 \times (7-7) + 10$ (308)

$T_7 =$ number of balloons in 7 th term of game $= 61$ (309)

2(via) to find the step the player would in level 5 if $T_n = 86$ is obtained as:

$T_n = 86, l = 5$ (310)

Migration level factor of player in level 5

$a = l - 1 = 5 - 1 = 4$ (311)

Migration step factor of player in level 5 =

$m = \frac{a^2+a+2}{2}$ (312)

Migration step factor of player in level 5

$m = \frac{4^2+4+2}{2}$ (313)

Migration step factor of player in level 5

$m = 11$ (314)

$T_n = k(a) + i(n-m) + f$ (315)

$86 = 17 \times 4 + 8 \times (n-11) + 10$ (316)

$8 \times (n-11) = 86 - 10 - 68$ (317)

$8 \times (n-11) = 8$ (318)

$n = 12$ (319)

The stage of the game is 12 th term.

The step the player would in level 5 (if $T_n = 86$) =

$s = \frac{2n-l^2+3l-2}{2}$ (320)

$s = \frac{2\times12-5^2+3\times5-2}{2} = 6$ (321)

The step the player would be in step 6

2(vib) is the number of terms the player would be in in the game for level 5 and step 6 is 12 th term.

$n = 12$ (322)

2(vii) to produce Kifilideen's Structural Matrix Framework for the balloon blow–up video game, we have:

The migration level value of the game $= k = 17$; the migration step value of the game $= i = 8$; the first term value of the game $= f = 10$. So, Kifilideen's Structural Matrix Framework for the balloon blow–up video game for the first five levels is presented in Table 10.

Table 10. The balloon blow–up video game of question 2 for the first five levels.

	l_1	l_2	l_3	l_4	l_5
s_1	$T_1 = 10$				
s_2		$T_2 = 27$			
s_3		$T_3 = 35$	$T_4 = 44$		
s_4			$T_5 = 52$	$T_7 = 61$	
s_5			$T_6 = 60$	$T_8 = 69$	$T_{11} = 78$
s_6				$T_9 = 77$	$T_{12} = 86$
s_7				$T_{10} = 85$	$T_{13} = 94$
s_8					$T_{14} = 102$
s_9					$T_{15} = 110$

Table 10 indicates that the migration level value is 17 and the migration step value is 8. Also, from the Table 10, it is noted that the difference between the number of balloons in the last member in a level and the last member in the successive level is 25. This is obtained by the addition of the migration level value, k and the migration step value, i.

(3) A jewelry manufacturing company produced a series of gold rings of various masses, following Kifilideen's Generalized Matrix Progression Sequence of infinite terms to have various levels and steps of gold rings. If the masses of the gold ring developed in the $1^{st}, 7^{th}$ and 12^{th} terms of the Kifilideen's Structural Matrix Framework are $200\ g, 500 g$ and $640\ g$ respectively; present the masses of the gold rings designed for the first five levels in the Kifilideen's Structural Matrix Framework.

Solution

From the question we have:

$T_1 = f = 200\ g$ (323)

For 7^{th} term, $n = 7,\ T_7 = 500\ g$ (324)

Migration level factor of the gold ring in the 7^{th} term

$= a = \frac{-1+\sqrt{8n-7}}{2}$ (325)

Migration level factor of the gold ring in the 7^{th} term

$= a = \frac{-1+\sqrt{8\times7-7}}{2}$ (326)

Migration level factor of the gold ring in the 7^{th} term

$= a = \mathbf{3}$ (327)

Migration level factor of the gold ring in the 7^{th} term

$= a = 3$ (328)

Migration step factor of the gold ring in the 7^{th} term

$= m = \frac{a^2+a+2}{2}$ (329)

Migration step factor of the gold ring in the 7^{th} term=

$m = \frac{3^2+3+2}{2}$ (330)

Migration step factor of the gold ring in the 7^{th} term=

$m = 7$ (331)

Level of the gold ring in the 7^{th} term =

$l = a + 1 = 3 + 1 = 4$ (332)

Step of the gold ring in the 7^{th} term =

$s = n - m + l = 7 - 7 + 4 = 4$ (333)

Position of the staff in the 7^{th}term =

$p = \frac{2n-a^2-a}{2}$ (334)

Position of the staff in the 7^{th} term =

$p = \frac{2\times7-3^2-3}{2}$ (335)

Position of the staff in the 7^{th} term =

$p = 1$ (336)

$T_n = k(a) + i(n - m) + f$ (337)

Seventh term

$= T_7 = k(3) + i(7 - 7) + f = 500\ g$ (338)

$3k + f = 500\ g$ (339)

Put (323) in (339)

$3 \times k + 200 = 500$ (340)

$3k = 300$ (341)

$k = 100\ g$ (342)

The migration level value of the gold ring =

$k = 100\ g$ (343)

For 12^{th} term, $n = 12,\ T_7 = 640\ g$ (344)

Migration level factor of the gold ring in the 12^{th} term

$= a = \frac{-1+\sqrt{8n-7}}{2}$ (345)

Migration level factor of the gold ring in the 12^{th} term

$= a = \frac{-1+\sqrt{8\times12-7}}{2}$ (346)

Migration level factor of the gold ring in the 12^{th} term

$= a = \mathbf{4}.2170$ (347)

Migration level factor of the gold ring in the 12^{th} term

$= a = 4$ (348)

Migration step factor of the gold ring in the 12^{th} term

$= m = \frac{a^2+a+2}{2}$ (349)

Migration step factor of the gold ring in the 12^{th} term

$= m = \frac{4^2+4+2}{2}$ (350)

Migration step factor of the gold ring in the 12^{th} term

$= m = 11$ (351)

Level of the gold ring in the 12^{th} term =

$l = a + 1 = 4 + 1 = 5$ (352)

Step of the gold ring in the 12 term =

$s = \frac{2n-l^2+3l-2}{2} = \frac{2\times 12-5^2+3\times 5-2}{2} = 6$ (353)

Position of the staff in the 12^{th} term =

$p = n - m + 1$ (354)

Position of the staff in the 12 term =

$p = 12 - 11 + 1 = 2$ (355)

Position of the staff in the 12^{th} term =

$p = 2$ (356)

$T_n = k(a) + i(n - m) + f$ (357)

$f = 200\ g$ and $k = 100$ (358)

Twelfth term =

$T_{12} = 100\ \times (4) + i(12 - 11) + 200 = 640\ g$ (359)

$400 + i + 200\ = 640\ g$ (360)

The migration step value of the gold ring =

$i = 40\ g$ (361)

To produce the masses of the gold rings designed for the first five levels in Kifilideen's Structural Matrix Framework; we have:

The migration level value of the mass of the gold ring = $k = 100\ g$; the migration step value of the mass of the gold ring = $i = 40\ g$ and the first term of the mass of the gold ring $= f = 200\ g$. So, Kifilideen's Structural Matrix Framework for the masses of the gold ring for the first five levels is presented in Table 11.

Table 11. The masses of the gold ring of question 3

	l_1	l_2	l_3	l_4	l_5
s_1	$T_1 = 200\ g$				
s_2		$T_2 = 300\ g$			
s_3		$T_3 = 340\ g$	$T_4 = 400\ g$		
s_4			$T_5 = 440\ g$	$T_7 = 500\ g$	
s_5			$T_6 = 480\ g$	$T_8 = 540\ g$	$T_{11} = 600\ g$
s_6				$T_9 = 580\ g$	$T_{12} = 640\ g$
s_7				$T_{10} = 620\ g$	$T_{13} = 680\ g$
s_8					$T_{14} = 720\ g$
s_9					$T_{15} = 760\ g$

Table 11 indicates that the migration level value is $100\ g$ and the migration step value is $40\ g$. Also, from the Table 11, it is noted that the difference between the mass of the gold ring in the last member in a level and the last member in the successive level is $140\ g$. This is obtained by the addition of the migration level value, k and the migration step value, i.

(4) A private college school implements Kifilideen's Generalized Matrix Progression Sequence of the infinite terms to develop salary structure for the staff [principal (level 1), vice principals (level 2), Head of Departments (level 3), Senior teachers I (level 4), Senior teachers II (level 5), Junior teachers I (level 6), Junior teachers II (level 7), Security Officer (level 8) and Cleaner] of the school. A Junior Teacher I (level 7) in step 11 receives a monetary incentive of ₦160, 000 while a Senior Teacher I (level 4) in position 4 receives a monetary incentive of ₦ 350, 000. A staff at the 43^{rd} term of the Kifilideen's Structural Matrix Framework receives monetary incentives of ₦ 40, 000. Determine the following:

(i) the level, the step, and the position of the staff that received monetary incentives of ₦ 40, 000;

(ii) the migration level value of the salary structure;

(iii) the migration step value of the salary structure;

(iv) the salary received by the principal (level 1) in step 1;

(v) the salary received by the Head of Department (level 3) in step 2;

(vi) produce Kifilideen's Structural Matrix Framework for the salary structure of the private school.

Solution

4(i) For 43^{rd} term, $n = 43, \quad T_{43} = ₦\,40,000$ (362)

The migration level factor, a of the staff in the 43^{rd} the term is achieved as:

Migration level factor of the staff in the 43^{rd} term =

$$a = \frac{-1+\sqrt{8n-7}}{2} \quad (363)$$

Migration level factor of the staff in the 43^{rd} term =

$$a = \frac{-1+\sqrt{8\times 43-7}}{2} \quad (364)$$

Migration level factor of the staff in the 43^{rd} term =

$$a = \mathbf{8}.6788 \quad (365)$$

Migration level factor of the staff in the 43^{rd} term =

$$a = 8 \quad (366)$$

So, the migration level factor, a of the staff in the 43^{rd} term is 8.

For the migration step factor, m of the staff in the 43^{rd} term, we have:

Migration step factor of the staff in the 43^{rd} term =

$$m = \frac{a^2+a+2}{2} \quad (367)$$

Migration step factor of the staff in the 43^{rd} term =

$$m = \frac{8^2+8+2}{2} \quad (368)$$

Migration step factor of the staff in the 43^{rd} term =

$$m = 37 \quad (369)$$

So, the migration step factor, m of the staff in the 43^{rd} term is 37.

The migration level factor, a of the staff in the 43^{rd} term obtained in (366) is used to obtain the value of the level of the staff in the 43^{rd} term which is presented as follows:

Level of the staff in the 43^{rd} term =

$$l = a + 1 = 8 + 1 = 9 \quad (370)$$

So, the staff in 43^{rd} term is in level 9.

The value of the level of the staff in the 43^{rd} term attained in (370), the migration step factor of the staff in the 43^{rd} term obtained in (369) and the number of terms of the staff in the 43^{rd} term is used to determine the value of the step of the staff in the 5^{th} term which is obtained as follows:

Step of the staff in the 43^{th} term =

$$s = n - m + l \quad (371)$$

Step of the staff in the 43^{rd} term =

$$s = 43 - 37 + 9 \quad (372)$$

Step of the staff in the 43^{rd} term $= 15$ (373)

Therefore, the staff in the 43^{rd} term is in step 15.

The number of terms and the migration step factor, m of the staff in the 43^{rd} term is used to determine the value of the position of the staff in the 43^{rd} term which is presented as follows:

Position of the staff in the 43^{rd} term=

$$p = n - m + 1 \quad (374)$$

Position of the staff in the 43^{rd} term =

$$p = 43 - 37 + 1 \quad (375)$$

Position of the staff in the 43^{rd} term = $p = 7$ (376)

So, the staff in the 43^{rd} term is in position 7.

From (370), (373) and (376), the staff in the 43^{rd} term is in level 9, step 15 and position 7 in Kifilideen's Structural Matrix Framework

4(ii) The equation generated from the 43^{rd} term using the Kifilideen's term formula of the Kifilideen's Generalized matrix progression sequence is presented as follows:

$$T_n = k(a) + i(n-m) + f \quad (377)$$

Number of term, n of the staff in the 43^{rd}term is 43, the migration level factor, a of the staff in the 43^{rd} term is 8 and the migration step factor, m of the staff in the 43^{rd} term is 37. From the question [4] the staff in the 43^{rd} term received monetary incentives of $₦\,40,000,$ so the equation generated for the 43^{rd}term is given as:

Forth – third term =

$$T_{43} = k(8) + i(43-37) + f = ₦\,40,000 \quad (378)$$

$$8k + 6i + f = ₦\,40,000 \quad (379)$$

For a junior teacher I (level 7) in step 11 receiving monetary incentives of ₦ 160, 000, we have:

The equation generated for a junior teacher I (level 7) in step 11 using the Kifilideen's term formula of the

Kifilideen's generalized matrix progression sequence is presented as follows:

$$T_n = k(l-1) + i(s-l) + f \quad (380)$$

For $l = 7, s = 11,$ we have: (381)

$$T_n = k(7-1) + i(11-7) + f = ₦\,160,000 \quad (382)$$

$$6k + 4i + f = ₦\,160,000 \quad (383)$$

For a senior teacher I (level 4) in position 4 receiving monetary incentives of ₦ 350, 000, we have:
The equation generated for a senior teacher I (level 4) in position 4 using the Kifilideen's term formula of the Kifilideen's generalized matrix progression sequence is presented as follows:

$$T_n = k(l-1) + i(s-l) + f \quad (384)$$

For $l = 4, s = 4,$ we have: (385)

$$T_n = k(4-1) + i(4-4) + f = ₦\,350,000 \quad (386)$$

$$3k + f = ₦\,350,000 \quad (387)$$

From (379), (383) and (387), we have:

$$8k + 6i + f = ₦\,40,000 \quad (378)$$

$$6k + 4i + f = ₦\,160,000 \quad (379)$$

$$3k + f = ₦\,350,000 \quad (380)$$

Subtract (380) from (378), we have:

$$5k + 6i = -₦\,310,000 \quad (381)$$

Subtract (379) from (378), we have:

$$2k + 2i = -₦\,120,000 \quad (382)$$

Multiply (382) by -3, we have:

$$-6k - 6i = ₦\,360,000 \quad (383)$$

Add (381) to (383), we have:

$$-k = ₦\,50,000 \quad (384)$$

$$k = -₦\,50,000 \quad (385)$$

Put (385) in (380), we have:

$$3k + f = ₦\,350,000 \quad (386)$$

$$3(-₦\,50,000) + f = ₦\,350,000 \quad (387)$$

$$f = ₦\,500,000 \quad (388)$$

Put (385) and (388) in (379), we have:

$$6k + 4i + f = ₦\,160,000 \quad (389)$$

$$6(-₦\,50,000) + 4i + ₦\,500,000 = ₦\,160,000 \quad (390)$$

$$4i = ₦\,160,000 + ₦\,300,000 - ₦\,500,000 \quad (391)$$

$$4i = -₦\,40,000 \quad (392)$$

$$i = -₦\,10,000 \quad (393)$$

So, the migration level value of the salary structure=

$$k = -₦\,50,000. \quad (394)$$

4(iii) the migration step value of the salary structure

$$= i = -₦\,10,000. \quad (395)$$

4(iv) the salary received by the principal (level 1) in step

$$1 = f = ₦\,500,000 \quad (396)$$

4(v) the salary received by the Head of Departments (level 3) in step 2 is determined as follows:

$$T_n = k(l-1) + i(s-l) + f \quad (395)$$

$l = 3, s = 2, k = -₦\,50,000, i = -₦\,10,000$ and

$$f = ₦\,500,000 \quad (396)$$

$$T_n = -₦\,50,000 \times (3-1) + (-₦\,10,000) \times (3-2) + ₦\,500,000 \quad (397)$$

$$T_n = -₦\,100,000 - ₦\,10,000 + ₦\,500,000 = ₦\,390,000 \quad (398)$$

The salary received by the Head of Departments (level 3) in step 2 is $₦\,390,000$.

(4vi) To produce Kifilideen's Structural Matrix Framework for the salary structure of the private school for the nine levels, we have:

The migration level value of the salary structure = $k = -₦\,50,000$; the migration step value of the salary structure = $i = -₦\,10,000$ and the first term $= f = ₦\,500,000$. So, Kifilideen's Structural Matrix Framework for the salary structure of the private school for the first nine levels is presented in Table 12. From Table 12, the principal (level 1) in step 1 received the highest monetary incentives with a value of ₦ 500,000. Also, the salary structure decreases across the levels and decreases within each level. The Table 12 indicates that the migration level value is $-₦\,50,000$ and the migration step value is $-₦\,10,000$. Also, from the Table 12, it is noted that the difference between the salaries received by the last member in a level and the last member in the successive level is -₦ 60,000. This is obtained by the addition of the migration level value, k and the migration step value, i.

Table 12. The salary structure of the private school of question 4 for the first nine levels.

	l_1	l_2	l_3	l_4	l_5	l_6	l_7	l_8	l_9
	Principal	Vice Principals	Head of Departments	Senior teachers I	Senior teachers II	Junior teachers I	Junior teachers II	Security Officer	Cleaner
s_1	₦ 500, 000								
s_2		₦ 450, 000							
s_3		₦ 440, 000	₦ 400, 000						
s_4			₦ 390, 000	₦ 350, 000					
s_5			₦ 380.000	₦ 340, 000	₦ 300, 000				
s_6				₦ 330, 00	₦ 290, 000	₦ 250, 000			
s_7				₦ 320, 000	₦ 280, 000	₦ 240, 000	₦ 200, 000		
s_8					₦ 270, 000	₦ 230, 000	₦ 190, 000	₦ 150, 000	
s_9					₦ 260, 000	₦ 220, 000	₦ 180, 000	₦ 140, 000	₦ 100, 000
s_{10}						₦210, 000	₦ 170, 000	₦ 130, 000	₦ 90, 000
s_{11}						₦ 200, 000	₦ 160, 000	₦ 120, 000	₦ 80, 000
s_{12}							₦ 150, 000	₦ 110, 000	₦ 70, 000
s_{13}							₦ 140, 000	₦ 100, 000	₦ 60, 000
s_{14}								₦ 90, 000	₦ 50, 000
s_{15}								₦ 80, 000	₦ 40, 000
s_{16}									₦ 30, 000
s_{17}									₦ 20,000

The difference between the salaries received by the last member in a level and the last member in the successive level $= k + i = (-₦\, 50,000\,) + (-₦\, 10,000) = -₦\, 60,000$ (399)

4. Conclusion

This study invented stepwise analysis, generation, and applications of Kifilideen's Matrix Structural Framework for an infinite term of increasing members of successive levels with the first level having one member of Kifilideen's Generalized Matrix Progression Sequence. The values of members of the cluster were designed and developed for Kifilideen's Generalized Matrix Progression Sequence of the infinite term of increasing members of successive levels with the first level having one member and Kifilideen's Structural Framework was generated for the clusters of such sequence. This Kifilideen's Matrix Structural Framework also helps to generate Kifilideen's formulas to identify members and assign values and grades of values to each member within and across levels of Kifilideen's Matrix Structural Framework. Applications of Kifilideen's formulas established for Kifilideen's Generalized Matrix Progression Sequence of the infinite term was carried out. Kifilideen's Matrix Structural Framework generated some help to exclusively differentiate varying members in the clusters into various levels and steps within levels.

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